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The osmotic condensational force  
of water vapor  
in the terrestrial atmosphere

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**Осмотическая сила конденсации водяного пара в земной атмосфере**

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**Аннотация**

В работе исследуется явление осмоса в смеси конденсирующихся и неконденсирующихся газов в условиях пространственного градиента температуры. Конденсация одного из газов смеси при понижении температуры в определенной области пространства приводит к падению давления смеси в этой области, возникновению силы и динамического потока смеси, направленного в эту область. Это явление представляет собой осмос, в котором роль полупроницаемой перегородки выполняет градиент температуры, приводящий к конденсации одного из газов смеси. Рассмотрена конденсационная теория ураганов и смерчей.

**Abstract**

Osmosis in the mixture of condensable and non-condensable gases in the presence of spatial temperature gradient is studied. Condensation of one of the mixture constituents caused by local temperature decrease results in the drop of total mixture pressure in this local area. The arising pressure-gradient force leads to the appearance of the dynamic gas flow directed towards the area where condensation takes place. This phenomenon represents osmosis of a particular type where the role of the semi-permeable membrane is played by the temperature gradient selectively removing, via condensation, one of the gases from the mixture. A condensational theory of hurricanes and tornadoes is developed.

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## 1. Introduction

The phenomenon of osmosis consists in the fact that partial pressures of particular constituents of gas mixtures or liquid solutions tend to spatial homogeneity independently of each other.

A special case of osmosis is represented by processes when the non-homogeneous spatial distribution of particular constituents of the gas mixture arises in the result of gas-to-liquid phase transition in some local area. Such phase transitions lead to thousand-fold volume decrease of the affected gas; this is equivalent to its local disappearance from the volume occupied by the mixture. The resulting spatial gradient of partial pressure of the condensing gas corresponds to a pressure-gradient force of osmotic nature. This force acts on a unit volume of gas mixture and is directed towards the area where condensation occurred. Under the action of the osmotic force gases of the mixture start to move dynamically towards the area where the phase transition is going on; the osmotic circulation of gases sets in.

Condensation of atmospheric water vapor that occurs in moist air in the presence of spatial temperature gradients invokes this type of osmotic circulation. The osmotic condensational force of water vapor and the resulting circulation are not related to the presence or absence of gravity; they are fundamentally different from the Archimedes force and the related adiabatic convection of ascending and descending air masses in the gravitational field of Earth. So far the osmotic circulation of gases under the action of osmotic force arising due to phase transitions remains unstudied and is not taken into account in the consideration of atmospheric circulation.

In this paper it is argued that the osmotic force of water vapor in presence of the observed temperature gradients and relative humidity is a major force determining the character of atmospheric circulation in the terrestrial atmosphere. The osmotic force of water vapor forms the basis of physical approach allowing one to quantitatively predict wind velocities observed in hurricanes and tornadoes, which, as in detail discussed in the paper, so far have not received a satisfactory theoretical explanation.

## 2. Aerostatic equilibrium of atmospheric air

Atmosphere is held at the planetary surface by the gravitational field. According to the kinetic theory of gases and Dalton's law (Landau et al., 1965; Landau, Lifshitz, 1987; Feynman et al., 1963) atmospheric gases come to equilibrium independently of one another (Feynman et al., 1963). Aerostatic equilibrium of atmospheric air arises when partial pressure  $p_i(z)$  of each  $i$ -th gas is balanced by the weight of the gas column above the

considered height  $z$ , and pressure change  $dp_i(z)$  over height increment  $dz$  is equal to weight of the gas in the layer of thickness  $dz$ :

$$-\frac{dp_i}{dz} = g\rho_i, \quad \rho_i \equiv M_i N_i, \quad (1)$$

where  $M_i$  is molar mass,  $N_i$  ( $\text{mol m}^{-3}$ ) is molar density and  $\rho_i$  is mass density of the  $i$ -th gas, respectively,  $g = 9.8 \text{ m s}^{-2}$  is the acceleration of gravity. Summing both sides of (1) over  $i$  and taking into account that  $\sum p_i = p$  and  $\sum \rho_i = \rho$ , where  $p$  and  $\rho$  are pressure and mass density of atmospheric air, respectively, we have

$$-\frac{dp}{dz} = g\rho. \quad (2)$$

Therefore, if each gas in the air mixture is in aerostatic equilibrium, the mixture as a whole is in aerostatic equilibrium as well. Equation (2) is equally valid for gases as well as for liquids and, for this reason, is often called the equation of hydrostatic equilibrium (Landau et al., 1965; Landau, Lifshitz, 1987; McEwan, Phillips, 1975; Tverskoi, 1951).

To specify that it is gas that is considered it is necessary to add the equation of state for the partial pressure of the  $i$ -th gas (Landau et al., 1965)

$$p_i = N_i RT \equiv g\rho_i h_i, \quad h_i \equiv \frac{RT}{M_i g}, \quad (3)$$

where  $R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$  is the universal gas constant. The value of  $g\rho_i h_i$  gives weight of atmospheric column of height  $h_i$  and density  $\rho_i$ . Combining (1) and (3) and solving them for  $p_i$  we have:

$$-\frac{dp_i}{dz} = \frac{p_i}{h_i}, \quad p_i(z) = p_{is} \exp\left\{-\int_0^z \frac{dz'}{h_i(z')}\right\}; \quad h_i \equiv \frac{RT}{M_i g}, \quad (4)$$

where low index  $s$  everywhere indicates corresponding values at the Earth's surface at  $z = 0$ . Summing (4) over  $i$  and taking into account that  $\sum N_i = N$  is molar density of air, we obtain equation of state for atmospheric air

$$p = NRT = g\rho h, \quad \rho \equiv MN, \quad h \equiv \frac{RT}{Mg}, \quad (5)$$

where  $M \equiv \sum M_i N_i / N$  is air molar mass. Equations (2) and (5) are formally similar to equations (1) and (3), but unlike (1) and (3) they do not yield an equation similar to equation (4) that follows from (1) and (3). This

is because  $N = \sum N_i$  and  $\rho = \sum M_i N_i$  are two different functions of molar density  $N_i$  that are not, unlike  $N_i$  and  $\rho_i$ , see (1), linearly related. Therefore, for atmospheric air as a whole aerostatic equilibrium is, instead of a single equation of the type of eq. (4), described by system of equations

$$p = \sum p_i, \quad -\frac{dp}{dz} = \frac{p}{h}, \quad M \equiv \sum \gamma_i M_i, \quad \gamma_i \equiv \frac{p_i}{p} = \frac{N_i}{N}, \quad (6)$$

where  $\gamma_i$  are the relative partial pressures of air gases. Due to different values of molar masses  $M_i$  of different air constituents and the related different scale heights of  $p_i$  vertical distributions (4), in aerostatic equilibrium volume ratios  $\gamma_i$  and air molar mass  $M$  depend on height  $z$ .

It is known from observations that in the atmosphere of Earth relative partial pressures  $\gamma_i$  for dry air constituents do not depend on height  $z$  up to about 90 km above sea level (McEwan, Phillips, 1975), i.e.  $\gamma_i(z) = \gamma_i(0)$ . For dry air as a whole (low index  $d$ ) one has  $\rho_d = NM_d = p_d/(gh_d)$ ,  $h_d \equiv RT/M_d g$ , where molar mass of dry air  $M_d = 29 \text{ g mol}^{-1}$  is height-independent. Therefore, for dry air equation (6) allows for a solution similar to (4)

$$-\frac{dp_d}{dz} = g\rho_d = \frac{p_d}{h_d}, \quad p_d(z) = p_{ds} \exp\left\{-\int_0^z \frac{dz'}{h_d(z')}\right\};$$

$$h_d \equiv \frac{RT}{M_d g}, \quad h_{ds} = 8.4 \text{ km}, \quad (7)$$

where the dependence of dry air scale height  $h_d(z)$  on  $z$  is, as in (3) and (4), exclusively determined by the  $z$ -dependence of air temperature  $T$ . Differential equation (7) formally coincides with the hydrostatic equilibrium equation (2). However, equation (7) and its solution do not coincide with the corresponding relations (4) for aerostatic equilibrium, when all dry air constituents have different scale heights of their vertical distributions over  $z$ . The observed coincidence of these distributions in the terrestrial atmosphere points unambiguously to the departure from aerostatic equilibrium caused by action of an additional force, which is considered in the sections to follow.

### 3. The osmotic evaporative force in the terrestrial atmosphere

At a given temperature concentration of water vapor cannot be higher than the maximum saturated concentration. At standard atmospheric pressure at the Earth surface the volume occupied by water vapor exceeds the volume of liquid water, to which it condenses, by three orders of magnitude.

Ignoring the liquid volume compared to that of gas and using the ideal gas equation of state for saturated water vapor,  $p_{\text{H}_2\text{O}} = N_{\text{H}_2\text{O}} RT$ , where  $p_{\text{H}_2\text{O}}$  is partial pressure,  $N_{\text{H}_2\text{O}}$  is molar density of saturated water vapor, respectively,  $T$  is absolute temperature, one can express the dependence of  $p_{\text{H}_2\text{O}}$  on  $T$  (Clausius-Clapeyron equation) as follows (Landau et al., 1965):

$$\frac{dp_{\text{H}_2\text{O}}}{p_{\text{H}_2\text{O}}} = T_{\text{H}_2\text{O}} \frac{dT}{T^2}, \quad T_{\text{H}_2\text{O}} \equiv \frac{Q_{\text{H}_2\text{O}}}{R} \approx 5300 \text{ K}, \quad (8)$$

where  $Q_{\text{H}_2\text{O}} \approx 44 \text{ kJ mol}^{-1}$  is molar latent heat of evaporation; its numerical value (8) corresponds to the global mean surface temperature. The tropospheric decrease of air temperature within the ten kilometers from the Earth surface does not increase the value of  $Q_{\text{H}_2\text{O}}$  by more than 5% (Landau et al., 1965), so  $Q_{\text{H}_2\text{O}}$  can be assumed to be temperature-independent.

Taking into account change of temperature with height  $z$ , as described by the temperature lapse rate  $\Gamma$  that is equal to the negative vertical temperature gradient,  $\Gamma \equiv -dT/dz$ , the dependence of saturated partial pressure of water vapor on height  $z$  (8) can be written as

$$-\frac{dp_{\text{H}_2\text{O}}}{dz} = \frac{p_{\text{H}_2\text{O}}}{h_{\text{H}_2\text{O}}}, \quad p_{\text{H}_2\text{O}} = h_{\text{H}_2\text{O}s} \left\{ -\int_0^z \frac{dz'}{h_{\text{H}_2\text{O}}(z')} \right\}, \quad h_{\text{H}_2\text{O}} \equiv \frac{T^2}{\Gamma T_{\text{H}_2\text{O}}}, \quad (9)$$

where low index  $s$  refers to the value of  $p_{\text{H}_2\text{O}}$  at the surface. The integral in (9) takes into account the dependencies of all variables,  $T$ ,  $\Gamma$  and  $T_{\text{H}_2\text{O}}$ , on height  $z$ . Under the approximation of constant  $T_{\text{H}_2\text{O}}$  (8), the value of  $h_{\text{H}_2\text{O}}$ , which is the scale height of the quasi-exponential vertical distribution of atmospheric water vapor (9), is unambiguously determined by the dependencies of absolute temperature  $T$  and, hence,  $\Gamma$ , on height  $z$ .

Differential equations for the  $z$  dependencies of saturated partial pressure of water vapor  $p_{\text{H}_2\text{O}}$  (9) and of water vapor in aerostatic equilibrium  $p_v$  ( $i = v$ ) (4) have identical form and both allow for exponential solutions. However, the expressions for  $h_{\text{H}_2\text{O}}$  (9) and  $h_v = RT/M_v g$ ,  $M_v = 18 \text{ g mol}^{-1}$ , which give the scale heights of the exponential drop of  $p_{\text{H}_2\text{O}}$  and  $p_v$ , respectively, are governed by different physical laws. It follows from formulae (4) and (9) that saturated water vapor can be in aerostatic equilibrium at a strictly specified dependence of temperature  $T$

and its lapse rate  $\Gamma \equiv -dT/dz$  on height  $z$  as stipulated by the following equality (Makarieva et al., 2006; Makarieva, Gorshkov, 2007):

$$h_{\text{H}_2\text{O}} = h_v = \frac{RT}{M_v g}, \quad \text{i.e.} \quad \frac{dT}{dz} = -\frac{T}{H}, \quad H \equiv \frac{RT_{\text{H}_2\text{O}}}{M_v g} = 250 \text{ km}. \quad (10)$$

Solving the obtained equation (10) for  $T(z)$  we have

$$T = T_s \exp\left(-\frac{z}{H}\right), \quad \Gamma \equiv -\frac{dT}{dz} = \Gamma_s \exp\left(-\frac{z}{H}\right) \approx \Gamma_{\text{H}_2\text{O}} = 1.2 \text{ K km}^{-1}. \quad (11)$$

Due to the large value of  $H$ , one can put  $\exp(-z/H) = 1$  for any  $z \leq h_{ds}$ , where  $h_{ds}$  describes the characteristic height of the atmosphere. In Eq. (11),  $\Gamma_{\text{H}_2\text{O}} = 1.2 \text{ K km}^{-1}$  is calculated for the mean global surface temperature  $T_s = 288 \text{ K}$  ( $15^\circ \text{C}$ ). Differences in the absolute surface temperatures of equatorial and polar regions change this value by no more than 10%.

The obtained value of  $\Gamma_{\text{H}_2\text{O}} = 1.2 \text{ K km}^{-1}$  (11) is a fundamental parameter dictating the character of atmospheric processes. At  $\Gamma < \Gamma_{\text{H}_2\text{O}}$  water vapor in the entire atmosphere is in aerostatic equilibrium, but it is saturated at the surface only, i.e.  $p_v(z) < p_{\text{H}_2\text{O}}(T(z))$  for  $z > 0$  and  $p_v(z) = p_{\text{H}_2\text{O}}(T_s)$  for  $z = 0$ , where  $p_v$  is partial pressure of water vapor at height  $z$  (and  $p_{\text{H}_2\text{O}}$ , as before, is the saturated pressure of water vapor at  $T(z)$ ). Relative humidity  $R_H \equiv p_v/p_{\text{H}_2\text{O}}$  decreases with height. As far as dry air is in the aerostatic equilibrium as well, at  $\Gamma < \Gamma_{\text{H}_2\text{O}}$  there are no macroscopic fluxes of either water vapor or air in the atmosphere. There is no input of water vapor and latent heat into the atmosphere from the hydrosphere or moist soil. Evaporation, i.e. the upward flux of water vapor from the surface to the atmosphere, is zero at any surface temperature. Solar radiation absorbed by the Earth's surface makes water evaporate from the oceanic and soil surface, but the evaporated water undergoes condensation immediately at a microscopic distance above the surface, which is of the order of one free path length of water vapor molecules. Energy released during this condensation is converted to thermal radiation of the Earth's surface.

At  $\Gamma > \Gamma_{\text{H}_2\text{O}}$  water vapor can be saturated in the entire atmosphere,  $p_v(z) = p_{\text{H}_2\text{O}}(T(z))$ , but it cannot be in aerostatic equilibrium. At all heights, partial pressure of water vapor exceeds the weight of water vapor column above the considered  $z$  point. Change of water vapor partial pressure over height increment  $dz$  exceeds the weight of water vapor in the local layer of thickness  $dz$ . The resulting departure from equilibrium leads to the

appearance of an upward directed, water vapor related osmotic force  $f_E$  acting on a unit volume of moist air. Using the equation of state (3) for water vapor, force  $f_E$  can be written as

$$f_E \equiv -\frac{dp_v}{dz} - gp_v = -\frac{dp_v}{dz} - \frac{p_v}{h_v}. \quad (12)$$

For saturated water vapor  $p_v = p_{\text{H}_2\text{O}}$  we take into account Eq. (9), so force  $f_E$  (12) becomes

$$f_E = -\frac{dp_{\text{H}_2\text{O}}}{dz} - g\rho_{\text{H}_2\text{O}} = -\frac{dp_{\text{H}_2\text{O}}}{dz} - \frac{p_{\text{H}_2\text{O}}(z)}{h_v} = p_{\text{H}_2\text{O}}(z) \left( \frac{1}{h_{\text{H}_2\text{O}}} - \frac{1}{h_v} \right), \quad (13)$$

$$h_{v_s} = 13.5 \text{ km}, \quad h_{\text{H}_2\text{O}_s} = 2.4 \text{ km},$$

where  $p_{\text{H}_2\text{O}} = \rho_{\text{H}_2\text{O}} g h_v$ ,  $\rho_{\text{H}_2\text{O}} = N_{\text{H}_2\text{O}} M_v$ , and  $h_{\text{H}_2\text{O}}$  and  $h_v$  defined in (9), (10).

Using (9) and (11) and the observed value of air temperature lapse rate  $\Gamma_{ob} = 6.5 \text{ K km}^{-1}$  one can represent force  $f_E$  (13) for saturated water vapor as

$$f_E = \frac{p_{\text{H}_2\text{O}}}{h_{\text{H}_2\text{O}}} (1 - \alpha) \equiv \frac{\rho u_E^2}{2h_{\text{H}_2\text{O}}}, \quad u_E^2 \equiv 2gh\gamma_{\text{H}_2\text{O}}(1 - \alpha),$$

$$\gamma_{\text{H}_2\text{O}} \equiv p_{\text{H}_2\text{O}}(z)/p(z), \quad \alpha \equiv \frac{h_{\text{H}_2\text{O}}}{h_v} \leq \frac{\Gamma_{\text{H}_2\text{O}}}{\Gamma_{ob}} = 0.18, \quad \gamma_{\text{H}_2\text{O}} \equiv \frac{p_{\text{H}_2\text{O}}(z)}{p(z)}. \quad (14)$$

The first term in the equality (14) is the major one. It represents the upward-directed pressure gradient force, see (13), for saturated water vapor. This term does not depend on the latent heat of vaporization  $Q_{\text{H}_2\text{O}}$  (8). The second relative term  $\alpha$  is a minor one; it accounts for the weight of water vapor column and contains  $Q_{\text{H}_2\text{O}}$  in the denominator. The magnitude of this term, see (8), (9), (11) does not exceed 18% and can be safely neglected in all estimates. We conclude that the osmotic force  $f_E$  of water vapor practically does not depend on the value of vaporization constant  $Q_{\text{H}_2\text{O}}$  at the observed  $\Gamma_{ob} = 6.5 \text{ K km}^{-1}$ . At global mean surface temperature of  $15^\circ\text{C}$  we have  $\gamma_{\text{H}_2\text{O}_s} = 0.02$ , so that from (7) and (14) we obtain  $u_E = 52 \text{ m s}^{-1}$ .

In the absence of aerostatic equilibrium, under the action of the osmotic force  $f_E$  atmospheric circulation is induced, which continuously mixes air masses with different water vapor contents. In the result, relative humidity  $R_H \equiv p_v/p_{\text{H}_2\text{O}}$  in the lower atmosphere becomes less than unity. Global mean value of relative humidity at the surface,  $R_H = R_{H_s}$ , is about 80%



(Held, Soden, 2000). Water vapor reaches saturation at some height  $z_H > 0$ . This height can be approximately estimated assuming  $f_E = 0$  in the absence of condensation at  $R_H < 1$  and putting  $p_v \equiv p_{\text{H}_2\text{O}} R_H$  into (12), to obtain, see (13):

$$\frac{1}{R_H} \frac{dR_H}{dz} = \frac{1}{h_{\text{H}_2\text{O}}} - \frac{1}{h_v} \equiv \frac{1}{h_H}, \quad h_H \approx 2.9 \text{ км}, \quad R_H(z_H) = 1,$$

$$R_H(z) = R_H(0) \exp\left(\frac{z}{h_H}\right), \quad \ln R_H(0) + \frac{z_H}{h_H} = 0.$$

Putting  $R_H(0) = 0.8$ ,  $-\ln R_H(0) = 0.2$ , we have  $z_H = 0.2 \times h_H \approx 600$  m. Starting from this height up, force  $f_E$  at  $R_H(0) = R_{H_s} < 1$  is expressed by the same formula (14) even under the assumption that at  $z < z_H$  we have  $f_E = 0$ . (In reality, in the aerodynamic equilibrium below height  $z_H$  as well neither water vapor is in aerostatic equilibrium, nor force  $f_E$  is zero.) Consequently,  $R_H(0) < 1$  exerts practically no impact on the magnitude of the osmotic force  $f_E$  in the atmospheric column.

The fact that, according to observations, molar mass  $M_d$  of dry air ( $M_d = \sum_i \gamma_{id} M_i = 29 \text{ g mol}^{-1}$ , where relative partial pressures  $\gamma_{id} \equiv p_i/p_d$ ,  $i \neq v$ , stand for all air constituents except water vapor and do not change with height) does not depend height  $z$ , indicates that partial pressures of all dry air constituents have one and the same distribution over height, the one described by Eq. (7). For each particular gas, its departure from the aerostatic equilibrium distribution (4) leads to the appearance of an osmotic force acting on air unit volume of moist air. Similar to (12), the osmotic force  $f_i$  related to the  $i$ -th gas is equal to

$$f_i = -\frac{dp_i}{dz} - g\rho_i = -\frac{dp_i}{dz} - \frac{p_i}{h_i} = p_i \left( \frac{1}{h_d} - \frac{1}{h_i} \right) = \frac{p_d}{h_d} \gamma_{id} (1 - \beta_i),$$

$$\gamma_{id} \equiv \frac{p_i}{p_d}, \quad \beta_i \equiv \frac{h_d}{h_i} = \frac{M_i}{M_d}, \quad \sum_i \gamma_{id} = 1, \quad \sum_i \gamma_{id} \beta_i = 1. \quad (15)$$

Summing the osmotic forces for all dry air constituents and using the last two sum rules in (14) we conclude that the cumulative osmotic force for all these gases is equal to zero, see (7):

$$\sum_{i \neq v} f_i = -\frac{dp_d}{dz} - g\rho_d = \frac{p_d}{h_d} \left( \sum_i \gamma_{id} - \sum_i \gamma_{id} \beta_i \right) = 0. \quad (16)$$

Therefore, the only uncompensated osmotic force remains the water vapor osmotic force  $f_E$  (12)-(14) sustained by the process of air moisture condensation. The constancy of  $M_d$  over  $z$  (14) and the resulting relation (15) are consequences of the existing dynamic fluxes of air driven by an external force. This role is played by the osmotic force  $f_E$  of water vapor. Note that the osmotic forces (15) for atmospheric nitrogen ( $i = N_2$ ) and oxygen ( $i = O_2$ ),  $f_{O_2}$  and  $f_{N_2}$ , that are the largest by their absolute magnitude among the dry air osmotic forces, are three times smaller than  $f_E$ . The sum of these two forces that are opposite in sign,  $f_{N_2} + f_{O_2}$ , is about one twentieth of  $f_E$  only.

If evaporation ceases and there is no input of water vapor from the surface into the atmosphere, at  $\Gamma > \Gamma_{H_2O}$  all water vapor condenses and precipitates, water vapor concentration and force  $f_E$  (12)-(14) turn to zero, the dry air becomes static. It is therefore logical to term the osmotic force  $f_E$  as the evaporative force. We emphasize that the process of evaporation is conditioned by the aerostatic non-equilibrium of water vapor; it develops maximum intensity at the maximum value of the evaporative force  $f_E$  that is realized in the atmospheric column fully saturated with water vapor.

#### 4. Ascending air fluxes induced by the evaporative force

The upward transport of air masses that arises in the presence of the evaporative force is of two types, dynamic and eddy. First, there are dynamic fluxes  $F_w$  of air and water vapor that ascend with a vertical velocity  $w$  under the action of the evaporative force (14). Second, there are turbulent eddy fluxes  $F_e$  caused by eddy diffusion. Eddy diffusion in the atmosphere arises in the presence of the evaporative force, which is the only uncompensated osmotic force acting on moist air.

As is well-known, eddy kinematic viscosity  $\nu_e$  can be determined from the linear scale of the largest eddy and linear velocity of air movement within it (Landau, Lifshitz, 1987). Therefore, for atmospheric air, which ascends with velocity  $w$  in the presence of the evaporative force acting on linear scale  $h_{H_2O}$ , eddy viscosity becomes  $\nu_e = cw h_{H_2O}$ , where  $c$  is a dimensionless multiplier of the order of unity (Landau, Lifshitz, 1987).

Dynamic flux of water vapor, which ascends together with other air gases with velocity  $w$ , is equal to

$$F_{H_2Ow} = wN_{H_2Os} = w\gamma_{H_2Os}N_s, \quad (17)$$

where  $N_{H_2Os}$  is saturated water vapor concentration at the surface.

Turbulent flux of water vapor  $F_{\text{H}_2\text{O}e}$  is equal to

$$F_{\text{H}_2\text{O}e} = -v_e \left[ \frac{dN_{\text{H}_2\text{O}}}{dz} - \left( \frac{dN_{\text{H}_2\text{O}}}{dz} \right)_0 \right], \quad (18)$$

where  $\left( \frac{dN_{\text{H}_2\text{O}}}{dz} \right)_0$  is the concentration gradient of water vapor that is saturated in the considered point  $z$  and is in aerostatic equilibrium, see (1), (2). Using the equation of state for saturated water vapor,  $p_{\text{H}_2\text{O}} = N_{\text{H}_2\text{O}} RT$ , relationships (13), (14) and the equality  $v_e = cw h_{\text{H}_2\text{O}}$  (derivatives of air temperature over  $z$  are present in both terms in (18) and cancel each other), we obtain the following expression for the eddy flux of water vapor at the surface:

$$F_{\text{H}_2\text{O}e} = \frac{v_e}{h_{\text{H}_2\text{O}}} \gamma_{\text{H}_2\text{O}s} N_s \left( 1 - \frac{h_{\text{H}_2\text{O}}}{h_v} \right) = \frac{v_e}{h_{\text{H}_2\text{O}}} \gamma_{\text{H}_2\text{O}s} N_s \times 0.82 = 0.82 F_{\text{H}_2\text{O}w}. \quad (19)$$

Thus, the dynamic and turbulent fluxes of water vapor practically coincide. At  $c = 1.2$  and  $v_e = 1.2wh_{\text{H}_2\text{O}}$  the equality is exact,  $F_{\text{H}_2\text{O}e} = F_{\text{H}_2\text{O}w}$ .

The obtained equation for eddy diffusion (19) offers explanation for the empirical observation (see, e.g., Chapter 4 in Lorenz, 1967) that there are no regions where water vapor transport would be downward directed, irrespective of the ascending or descending dynamic movement of air masses in the considered regions. In regions where evaporation is high and the evaporative force is large, masses of moist air ascend. According to the law of matter conservation (the continuity equation), in the regions with lower evaporation these air masses should descend. As discussed above, see (7), (16), for dry air as a whole there is no departure from the hydrostatic equilibrium. Eddy fluxes related to the departure from aerostatic equilibrium of individual dry air constituents (see Section 5 below) are small and would demand special high-precision measurements for their observation. Turbulent mixing of dry air does not therefore produce directional dynamic air fluxes. Such fluxes are dictated by the large-scale dynamic air movement with vertical velocity  $w$  and horizontal velocity  $u$ . Atmospheric water vapor is out of aerostatic equilibrium everywhere, irrespective of the magnitude of regional evaporation. Eddy viscosity  $v_e$  is a positive magnitude depending on the absolute value of vertical velocity  $w$ ,  $v_e = c|w|h_{\text{H}_2\text{O}}$ . Therefore, within the descending dynamic flux of moist air, the upward-directed eddy flux of water vapor can fully compensate the

descending dynamic flux of water vapor,  $F_{\text{H}_2\text{O}}^\downarrow = F_{\text{H}_2\text{Ow}}^\downarrow + F_{\text{H}_2\text{Oe}}^\uparrow =$   
 $= F_{\text{H}_2\text{Ow}}^\uparrow - |F_{\text{H}_2\text{Oe}}^\uparrow| \approx 0$ . This agrees with observations (Lorenz, 1967) and  
justifies the choice of  $\nu_e = 1.2|w|h_{\text{H}_2\text{O}}$ .

In the stationary case the total upward flux of water vapor from near the  
Earth's surface to the atmosphere,  $F_{\text{H}_2\text{O}} \equiv F_{\text{H}_2\text{Ow}} + F_{\text{H}_2\text{Oe}}$ , has to be  
compensated by an equal flux  $F$  of water vapor input into the considered  
area at the Earth's surface. If this input flux is determined by the magnitude  
of local evaporation  $E$ , then, taking into account the approximately equal  
magnitudes of the dynamic and diffusional components of  $F_{\text{H}_2\text{O}}$ , the  
stationary velocity of ascending air movement will be  $w = E/(2N_{\text{H}_2\text{O}})$ .

Moisture evaporated from the Earth's surface rises up to the atmosphere  
under the action of the osmotic force  $f_E$  (12) and returns to the surface after  
condensation and precipitation. Movement of air masses should follow  
closed trajectories, which particular forms depend on boundary conditions.

For the global mean value of evaporation  $\bar{E}$ , which coincides with the  
global mean precipitation  $\bar{P}$ ,  $\bar{P} = 55 \times 10^3 \text{ mol H}_2\text{O m}^{-2} \text{ year}^{-1}$  (L'vovitch,  
1979) and water vapor concentration  $N_{\text{H}_2\text{O}} = 0.7 \text{ mol m}^{-3}$   
( $\gamma_{\text{H}_2\text{O}s} \equiv \gamma_{\text{H}_2\text{O}}(0) = 0.02$ ), which is saturated at the surface at global mean  
surface temperature  $T = 288 \text{ K}$ , we have:

$$w = E/(2N_{\text{H}_2\text{O}}) = 1.3 \text{ mm s}^{-1}. \quad (20)$$

Thus, for the global mean eddy viscosity we obtain  $\bar{\nu}_e \approx 1.2\bar{w}N_{\text{H}_2\text{O}} \approx$   
 $1.2 \times 1.3 \text{ mm s}^{-1} \times 2.4 \text{ km} \approx 3.7 \text{ m}^2 \text{ s}^{-1}$ . This magnitude comes close to the  
phenomenological value taken to be  $3.5 \text{ m}^2 \text{ s}^{-1}$  in modeling studies (Fang,  
Tung, 1999). This is another justification for the assumed equality  
 $\nu_e = 1.2|w|h_{\text{H}_2\text{O}}$ .

Mean velocity  $\bar{\nu}$  of  $\text{H}_2\text{O}$  molecules leaving the liquid phase and  
moving into the upper hemisphere coincides, in thermal equilibrium, with  
mean thermal velocity of air molecules (Feynman et al., 1963). At  $\Gamma < \Gamma_{\text{H}_2\text{O}}$   
at a distance of the order of one free path length of air molecules ( $\sim 10^{-7} \text{ m}$ )  
from the Earth's surface the motion of molecules moving upward from the  
liquid phase equilibrates with the motion of air molecules; vertical velocity  
component of the evaporated  $\text{H}_2\text{O}$  molecules becomes zero. Numbers of  
 $\text{H}_2\text{O}$  molecules leaving the gas phase for the liquid, and vice versa, per unit

time, become equal. In this case the flux of water vapor from the hydrosphere to the atmosphere, i.e. evaporation, is equal to zero.

### 5. Eddy diffusion and the constancy of dry air molar mass

Within the dynamic ascending or descending air masses, gases of air additionally participate in eddy diffusion. Eddy flux of a particular gas is proportional to the departure of its concentration from the equilibrium value, with the proportionality coefficient equal to eddy viscosity (eddy diffusivity)  $\nu_e$ . For each gas, eddy diffusion tends to restore the aerostatic equilibrium (4). However, the resulting osmotic eddy fluxes of concentrations for the major air constituents appear to be dozens of times smaller as compared to the dynamic flux of air as a whole, the latter maintained by the evaporative force. Indeed, eddy flux  $F_{ei}$  of concentration of the  $i$ -th gas is equal to, see (15),

$$F_{ei} = -\nu_e \left[ \frac{dN_i}{dz} - \left( \frac{dN_i}{dz} \right)_0 \right] = \nu_e N_i \left( \frac{1}{h_d} - \frac{1}{h_i} \right) = \frac{\nu_e}{h_d} N_i (1 - \beta_i), \quad (21)$$

$$\beta_i \equiv \frac{h_d}{h_i},$$

where it is taken into account that  $N_i = p_i/RT$ , while  $(dN_i/dz)_0$  is the aerostatic equilibrium concentration gradient. The contributions of the derivative of  $T$  over  $z$  are the same in the equilibrium and non-equilibrium terms in (21) and cancel each other, as in (18). As in Eq. (16) above, sum of the osmotic eddy fluxes of all dry air constituents (21) is zero,  $\sum F_{ei} = 0$ . In other words, in observations not telling apart the different dry air constituents, these fluxes cannot be registered.

Using the equality  $\nu_e \approx wh_{\text{H}_2\text{O}}$ , for the ratio  $\varepsilon_i \equiv F_{ei} / F_{wi}$ , where  $F_{wi} = wN_i$  is the dynamic flux of the  $i$ -th gas maintained by the evaporative force (12), we have

$$\varepsilon_i = \frac{1 - \beta_i}{\beta_E}, \quad \beta_E \equiv h_d / h_{\text{H}_2\text{O}} \approx 3.5; \quad \varepsilon_{\text{N}_2} \approx 0.01, \quad \varepsilon_{\text{O}_2} \approx -0.03. \quad (22)$$

These estimates explain the observed approximate independence of the relative partial pressures of the major dry air constituents and dry air molar mass  $M_d$  on height  $z$ . According to (7) and (8), at constant  $M_d$  the relatively small eddy fluxes of oxygen with  $M_{\text{O}_2} > M_d$  are always directed upwards, while the eddy fluxes of nitrogen with  $M_{\text{N}_2} < M_d$  are always directed downwards. However, this does not lead to any considerable enrichment of

surface air with oxygen, or to a considerable deficit of nitrogen in the surface air, as far as the diffusional fluxes working to change concentrations of the gases, are compensated by corresponding changes in the much larger directional dynamic air fluxes maintained by the evaporative force.

Note that diffusional mixing, molecular as well as turbulent, should return the concentrations of dry air gases to their equilibrium aerostatic distributions (4). For this reason turbulent mixing cannot be considered as the cause of the observed constancy of the gaseous composition of dry air (Tverskoi, 1951; Glickman, 2000), i.e. of height-independence of ratios  $p_d/p \equiv \gamma_d$  and molar mass of dry air  $M_d$ , which is a non-equilibrium state (9). Turbulent mixing, as compared to molecular diffusion, restores the equilibrium many orders of magnitude more quickly.

Note also that introduction of mass-related gas constants in meteorology (Tverskoi, 1951; Glickman, 2000),  $R_d \equiv R/M_d$  и  $R_v \equiv R/M_v$ , instead of the universal gas constant  $R$  results in the loss of information about the main physical property of the ideal atmospheric gas, namely that the equations of state  $p = NRT$  for mixture as a whole and its individual constituents  $p_i = N_iRT$  do not depend either on the quantitative composition of mixture, i.e. ratios  $\gamma_i \equiv p_i/p$ , or on molecular masses  $M_i$  of particular gases in the mixture. Replacement of the universal gas constant by  $R_d \equiv R/M_d$  for dry air in the entire atmospheric column without investigating the physical reasons of why  $R_d$  is constant, is physically meaningless. So is the Similarly, introduction of the so-called virtual temperature, when the equation of state for moist air (5),  $p = \rho gh$ ,  $h \equiv RT/Mg$ ,  $M = \gamma_d M_d + \gamma_v M_v$ , i.e.  $1 = \gamma_d \beta_d + \gamma_v \beta_v$ ,  $\beta_i \equiv M_i/M$ , is equivalently re-written as  $p = \rho gh_v$ ,  $h_v \equiv R_d T_v/g$ ,  $R_d \equiv R/M_d$ , where  $T_v \equiv T [1 - \gamma_v(1 - \beta_v)]^{-1}$  is virtual temperature accounting for the quantitative parameters of water vapor  $\gamma_v$  and  $\beta_v$ , is physically misleading. This representation formally allows one to consider moist air, as well as dry air, as being in hydrostatic equilibrium (Tverskoi, 1951; Glickman, 2000).

## 6. Evaporative pump of moist air

Evaporative force induces horizontal air fluxes, which, near the Earth's surface, propagate in the direction from areas with lower, to areas with higher, evaporation. Difference in the magnitudes of the evaporative force in these areas causes air to ascend with velocity  $\Delta w$  in areas with higher evaporation and to descend in areas with lower evaporation. In consequence, there must exist a horizontal air flux with velocity  $u$  from the area with lower, to the area with higher, evaporation to compensate the

opposite vertical air movements in the considered areas. Let us denote width of the border between the two areas as  $D$  and length of the area with higher evaporation as  $L$ . Flux of moist air through the border surface  $Dh$  between the two areas,  $\rho u Dh$ , must be equal to the upward flux of moist air,  $\rho \Delta w DL$ , ascending over the area  $DL$  of higher evaporation. This results in relationship  $\Delta w = uh/L$ .

The two areas having different evaporation  $E_1 > E_2$ , horizontal flux of moist air is directed from the “donor” area 2 to the “recipient” area 1. Horizontal  $u$  and vertical  $\Delta w$  velocities of air fluxes are related by the matter conservation law as  $u = \Delta w L/h$ , where  $L = hu/\Delta w$  is the linear size of the recipient area. Linear size of the donor area should be of the same order of magnitude.

In the stationary state of constant wind velocities the power of the evaporative pump exerted by the evaporative force and maintained by solar radiation should coincide with the power of friction forces. Total power of

the upward-directed evaporative force is  $\Delta w DL \int_0^{\infty} f_E(z) dz =$

$\rho(u_E^2/2) \Delta w DL$ , its value per unit surface area is  $\rho(u_E^2/2) \Delta w$ . Turbulent friction destroys the main horizontal air stream results into smaller eddies. Rotation velocity  $u_s$  and kinetic energy density  $\rho u_s^2/2$  of these eddies does not depend on velocity  $u$  of the main streamflow; instead, they are determined by the weight of atmospheric column and vertical size  $z_s$  of the roughness of the planetary surface (height of ocean waves, forest trees etc.). As the main stream passes along the surface, a new small eddy is formed every  $z_s/u$  seconds. This confines the total power of the turbulent friction force at the surface as  $\rho(u_s^2/2) (u/z_s) DL z_s$ , where  $DL z_s$  — is the volume where the friction force is acting and the small eddies are formed. For the power of turbulent friction force per unit surface area we have  $\rho(u_s^2/2) u$  (Kármán, Rubach, 1912; Kochin, Roze, 1932). Far above the surface the main horizontal streamflow is decelerated by the ascending flows having vertical velocity  $\Delta w$  that form eddies with linear size of the order of atmospheric scale height  $h$ . New eddies bud off from the main streamflow every  $h/u$  seconds. Total power of turbulent friction force far from the planetary surface is  $\rho(\Delta w^2/2) (u/h) DL h$ , where  $DL h$  is the atmospheric volume where this force acts and the large eddies are formed; per unit surface this power is  $\rho(\Delta w^2/2)u$ . Equating the powers of the evaporative

force and cumulative forces of turbulent friction and canceling the common multiplier  $(\rho/2)DL$  from the equality one obtains

$$u_E^2 \Delta w = u_s^2 u + (\Delta w)^2 u \approx u_s^2 u, \quad \Delta w = uh/L. \quad (23)$$

The second equality is the continuity equation; on its grounds the second term in (23), which contains the small magnitude  $(\Delta w)^2$  can be neglected in the first equality of (23).

Frictional velocity  $u_s$  (also denoted in the literature as  $u_*$ ) of the small eddies near the surface, which does not depend on  $u$  and is determined by weight of the atmospheric column, from dimensional (similarity) considerations can be written as

$$u_s^2 = C_1 g z_s. \quad (24)$$

Coefficient  $C_1$  (Froude number) can be deduced from empirical data (Charnock, 1955; Garratt, 1977; Businger, Businger, 2001). (Note that when the evaporative force  $f_E$  is taken into account, the conditions of neutral stratification for which the Kármán law holds (Monin, Obukhov, 1953), are not met.) Order-of-magnitude estimates of  $u_s^2$  and  $C_1$  can be obtained from (23) using the global mean values of  $\bar{u} = 7 \text{ m s}^{-1}$  for horizontal wind velocity (Gustavson, 1979),  $\Delta\bar{w} \approx 10^{-3} \text{ m s}^{-1}$  (20) and putting  $z_s \sim 0.3 \text{ m}$  as the mean wave height in the ocean:

$$u_s^2 = \Delta\bar{w} u_E^2 / \bar{u} = 0.42 \text{ m}^2 \text{ s}^{-2}, \quad u_s = 0.65 \text{ m s}^{-1}, \quad C_1 = 0.04. \quad (25)$$

Assuming the obtained value of  $C_1$  to be a universal constant we have

$$u = \Delta w \frac{u_E^2}{C_1 g z_s}, \quad L = h \frac{u}{\Delta w} = h \frac{u_E^2}{C_1 g z_s}. \quad (26)$$

Horizontal wind velocity  $u$  and the horizontal scale length  $L$  of the atmospheric circulation driven by the evaporative pump are thus unambiguously determined by the magnitude of the evaporative force ( $u_E^2$ ), vertical velocity of the ascending air masses  $\Delta w$  and surface roughness height  $z_s$ . For global mean values of  $\Delta\bar{w} \sim 10^{-3} \text{ m s}^{-1}$ ,  $\bar{u} \sim 10 \text{ m s}^{-1}$ ,  $h \sim 10 \text{ km}$  we have  $L \sim 10^4 \text{ km}$ , i.e. a length scale coinciding with that of continents and oceans (Makarieva, Gorshkov, 2007).

So far the evaporative force has not been considered in the meteorology. For this reason there have been no physical magnitudes to equate the power of turbulent friction forces with, cf. (23). As a way out, the power of turbulent friction forces per unit surface area were equated to the value of  $C_D \rho u^3/2$ , where dimensionless coefficient  $C_D$  was termed drag coefficient. However,  $\rho u^3/2$  is not related to the dissipative power per unit



surface area, as it represents the kinetic power of the main horizontal streamflow per unit area of the vertical cross-section perpendicular to horizontal stream velocity  $u$ . Thus, coefficient  $C_D$  was determined from equating an arbitrarily chosen magnitude,  $\rho u^3/2$ , to the dissipative power  $\rho(u_s^2/2)u$  (23), i.e. it was assumed that  $C_D = u_s^2/u^2$  (see, e.g., Businger, Businger, 2001). Taking the evaporative force into account we obtain then from (23) that  $C_D = (u_E/u)^2(\Delta w/u) = (u_E/u)^2(h/L)$ . A dimensionless coefficient introduced in this way represents a formal mathematical change of variables and is physically meaningless. One could have similarly put  $C_D = u_s^4/u^4$  or  $C_D = (u_s/u)^n(h/L)^m$ , where  $n$  and  $m$  are arbitrary numbers.

Velocities of ascending air masses  $\Delta w$  can be unrelated to solar energy and the existence of a donor and recipient areas, but be, instead, determined by the store of water vapor accumulated in the atmospheric column. These velocities  $\Delta w = w \gg \bar{w}$  (22) can reach  $w \geq u_s \sim 1 \text{ m s}^{-1}$  and beyond. Then the second term in the right-hand side equality of (23), which we neglected, becomes of the order of or even larger than the first term (the one describing surface friction), while horizontal velocities approach  $u_E = 50 \text{ m s}^{-1}$ .

Such horizontal velocities are characteristic of hurricanes and tornadoes. Horizontal dimensions of these wind structures conform to the second equality in (23), which takes the form  $uh = wl$ . For the horizontal scale  $l$  of hurricanes we have  $l \sim 30h \sim 300 \text{ km}$ ,  $w \sim 0.03u \sim 1 \text{ m s}^{-1} \gg \bar{w}$  (20). For the horizontal dimension of tornadoes we have  $l \leq h \sim 10 \text{ km}$  and  $w \sim u \sim 50 \text{ m s}^{-1}$ . In these cases moisture evaporates on a large area  $L^2$  with a characteristic velocity of water vapor ascent (20) and undergoes condensation on a much smaller area  $l^2$  with characteristic vertical velocities  $w \geq 10^3 \bar{w}$ . The power of condensation (latent heat release) per unit surface area,  $N_{\text{H}_2\text{O}}Q_{\text{H}_2\text{O}}w$  ( $\text{W m}^{-2}$ ), is thousands of times greater than the power of evaporation under the action of solar radiation,  $\sim N_{\text{H}_2\text{O}}Q_{\text{H}_2\text{O}}\bar{w}$ , see (20),  $\bar{E} = Q_{\text{H}_2\text{O}}\bar{w}$ . Hurricanes and tornadoes arise in the regions of anomalously high condensation rates where the stationary drag of the large-scale osmotic evaporative air pump is absent.

## 6. Forest pump of atmospheric moisture

Air masses rising in the area with higher evaporation later descend in the area with lower evaporation. At the surface they are then enriched with moisture evaporated in that area and flow horizontally back to the area with higher evaporation to ascend there. In the course of the ascending motion of

air masses, moisture evaporated in the area with lower precipitation condenses and precipitates over the area with higher evaporation. This process of moisture transport occurs irrespective of temperature and moisture content differences between the two areas, provided the difference in evaporation rates persists. In particular, moisture can be transported from area with lower moisture content and lower evaporation to the area with higher moisture content and higher evaporation. Such process can be termed the evaporative pump of atmospheric moisture. Its power is determined by the available solar energy spent on evaporation and maintenance of the observed lapse rate of air temperature  $\Gamma_{ob}$ .

Due to the high leaf area index, which is defined as the total area of all leaves of the plant divided by the plant projection area on the ground surface, the cumulative evaporating surface of natural forest canopy can be several times larger than the open water surface of the same area. Evaporation from forest and the associated ascending fluxes of moist air can be several times greater than the corresponding magnitudes over the oceanic surface, approaching the maximum possible values affordable at a given flux of solar radiation. Forest evaporation corresponding to the global mean solar flux absorbed by the Earth's surface,  $I = 150 \text{ W m}^{-2}$  (Schneider, 1989), can thus reach a value of  $I/(\rho_{H_2O}Q) \approx 2 \text{ m year}^{-1}$ , where  $\rho_{H_2O} = 10^3 \text{ kg m}^{-3}$  is mass density of liquid water density and  $Q = 2.4 \times 10^6 \text{ kJ kg}^{-1}$  is latent heat of evaporation per unit mass. (Note that here evaporation flux is calculated as the rate at which the layer of liquid water diminishes, not as the ascending flux of water vapor, cf. (20). The obtained rate does not depend on surface temperature.) The observed mean global evaporation from the oceanic surface is substantially lower at about  $1.2 \text{ m year}^{-1}$  (L'vovitch, 1979), i.e. nearly twice as low as maximum forest evaporation. The ascending fluxes of moist air generated by forest evaporation induce the compensating low-level horizontal influx of moisture-laden air from the ocean with horizontal velocity  $u$  of the order of  $(2-10) \text{ m s}^{-1}$ . When the incoming air fluxes ascend, the oceanic moisture condenses and precipitates over the forest, thus compensating the gravitational river runoff (Makarieva, Gorshkov, 2007). Unburdened of moisture, dry air returns to the ocean from land in the upper atmosphere.

Forest moisture pump is peculiar in that a strict biological control is exerted over the process of evaporation. With its very large evaporative surface of leaves compared to open water surface, with its capability of regulating transpiration by regulating the degree of stomata opening/closure, forest is able to sustain evaporation and precipitation at an

optimal level determined by the biological soil moisture demand. The large value of tree height  $\sim z_s$  keeps the horizontal velocity  $u$  of air flow constant, excluding the possibility of hurricane and tornado formation. Year-round sustainability of precipitation and soil moistening results in the observed constancy of river runoff in undisturbed forest-covered river basins (Makarieva, Gorshkov, 2007). Pumping enormous amounts of atmospheric moisture from the ocean via the coastline, forest regulates precipitation to be spatially uniform over the entire river basin; moisture is then returned to the ocean in the liquid state as runoff. Forest control of precipitation prevents moisture shortage and droughts as well as the flood-pregnant excessive precipitation, particularly in the beginning of moisture way inland, where, near the coastline, the horizontal ocean-to-land moisture flux is the largest.

Let  $F(x)$  be the horizontal moisture flux equal to the amount of atmospheric moisture passing inland across a unit horizontal length perpendicular to the stream line per unit time at distance  $x$  from the ocean, dimension  $\text{kg H}_2\text{O m}^{-1} \text{s}^{-1}$ . For a river basin covered by natural forest we have

$$\frac{dF(x)}{dx} = R = \text{const}, \quad F(x) = F(0) - Rx, \quad F(0) = RL, \quad 0 \leq x \leq L, \quad (27)$$

where  $R$  is river runoff per unit surface area,  $L$  is the linear size of river basin, which, as estimated in the previous section, can reach  $10^4$  km. Large evaporative surface (with evaporation exceeding that of the open ocean) and large tree height (defining a constant optimal turbulent friction force and relatively low velocity for the horizontal air flow, not allowing for hurricane and tornado formation) are the two fundamental physical characteristics of natural forests that determine functioning of forest moisture pump.

Scarce and low vegetation with relatively small leaf surface area can develop evaporation fluxes exceeding those of the open ocean only under some special conditions, in particular, in summer when the available solar radiation is the largest. During such periods as, for example, during the so-called wet monsoon, flux  $F$  of moist air propagates from the ocean to land. However, in this case river runoff, which is proportional to precipitation that is in this case unregulated by the scarce vegetation, changes proportionally to flux  $F$ . This results in the exponential decline of runoff  $R(x)$  and precipitation with distance  $x$  from the ocean and in the excessive amount of precipitation (causing floods) in the vicinity of the coastline:

$$R(x) = \frac{dF(x)}{dx} = -\frac{1}{\lambda} F(x), \quad \frac{dF(x)}{dx} = \left( \frac{dF(x)}{dx} \right)_0 \exp\left(-\frac{x}{\lambda}\right), \quad (24)$$

where  $\lambda$  is the exponential scale length of precipitation decline in non-forested regions, which is of the order of 600 km on a global average (Makarieva, Gorshkov, 2007). The exponential decline of runoff and precipitation testifies for the inability of scarce and low vegetation to control runoff and precipitation.

When in winter season the solar radiation flux is minimal, oceanic evaporation exceeds the evaporation from non-forest vegetation. Accordingly, air at the surface is drawn from land to the ocean, as, for example, during the dry monsoon season. Notably, the rotation of wet and dry monsoon seasons is explained by the difference in solar radiation and by the changing sign of the difference between terrestrial and oceanic evaporation fluxes, not by the temperature differences between land and ocean surfaces. In deserts, where evaporation is virtually zero year round, air fluxes are invariably directed from land to the ocean despite the changing sign of the difference between desert and oceanic temperatures. In the result, deserts appear to be locked for oceanic moisture year round.

Pressure difference  $\Delta p$  associated with the departure of water vapor distribution from aerostatic equilibrium and with the action of the evaporative force is equal to  $\Delta p \sim p_{\text{H}_2\text{O}}(0) \approx 10^{-2} p(0)$ , where  $p(0) = 10^5$  Pa is air pressure at the Earth's surface. This value describes pressure deficit in the region where evaporation from the surface occurs. The value of  $\Delta p$  coincides with mean characteristic horizontal pressure differences observed at the sea level within cyclones and anticyclones. The linear size of cyclones and anticyclones being of the order of  $10^3$  km, for the horizontal barometric gradient  $dp_s/dx$  associated with the evaporative force we obtain  $dp_s/dx \sim (10^3 \text{ Pa})/(10^3 \text{ km}) = 1 \text{ Pa km}^{-1} = 1 \text{ mbar (100 km)}^{-1}$ , which agrees well with the observed characteristic values of horizontal barometric gradient (Lorenz, 1967; McEwan, Phillips, 1975).

## 8. Conclusions

The evaporative force has escaped consideration by theoretical meteorology due to the physically incorrect assumption that moist atmosphere can on average exist in the state of hydrostatic equilibrium (Lorenz, 1967; Tverskoi, 1951). These ideas can be traced to the correctional terms of the barometric formula derived by Pierre-Simon Laplace in the beginning of the 18<sup>th</sup> century before the kinetic theory of gases was developed and without accounting for Dalton's law that was independently formulated at approximately the same time. Subsequent introduction of virtual temperature made all moist air equations formally identical to the

corresponding equations for dry air (Lorenz, 1967; Tverskoi, 1951). The same ideas served to preserve the term “hydrostatic equilibrium” in the meteorological literature (Lorenz, 1967; McEwan, Phillips, 1975; Tverskoi, 1951) preventing the usage of “aerostatic equilibrium”, the term responding to the physical essence of the phenomenon in question. The assumption of hydrostatic equilibrium of moist atmosphere was supported by incorrect interpretation and lack of quantitative theoretical analysis, see (21), (22), of the observed constancy of dry air composition in the troposphere (height-independence of molar mass of dry air), which was thought to be caused by turbulent air mixing (Lorenz, 1967; McEwan, Phillips, 1975; Tverskoi, 1951), see Section 5.

These views come in conflict with the modern kinetic theory of gases, which explains Dalton’s law and the aerostatic Boltzmann distribution at molecular level (Landau et al., 1965; Feynman et al., 1963). If moist air as a whole were in hydrostatic equilibrium, then the observed radical departure from equilibrium of atmospheric water would, in static air, have been compensated by an equivalent, yet of opposite sign, departure from equilibrium of dry air constituents. Such a situation is prohibited by Dalton’s law and would contradict the observed phenomena of osmosis.

Indeed, in aerostatic equilibrium equal numbers of molecules of each gas cross any horizontal plane in the upward and downward directions (Feynman et al., 1963). This statement forms the ground for theoretical explanations of both Dalton’s law, Boltzmann’s distribution and osmosis (Landau et al., 1965; Feynman et al., 1963). Deviation of any particular gas in the mixture from equilibrium brings about diffusion fluxes that tend to restore the equilibrium distribution of this particular gas. Eddy diffusion restores the equilibrium several orders of magnitude more rapidly than does molecular diffusion, see Section 5. Therefore, if one gas in a mixture deviates from equilibrium, this deviation cannot in principle be compensated by the reverse deviations of other gases from the equilibrium, so that the sum of these gases is in hydrostatic equilibrium. This statement is the essence of the theoretical explanation of the phenomenon of osmosis.

As we showed, evaporation, i.e. flux of water vapor from the Earth’s surface to the atmosphere, is zero at any surface temperature if the vertical lapse rate of air temperature is less than the critical value of  $1.2 \text{ K km}^{-1}$ . At higher lapse rate there arises the osmotic force of water vapor (termed evaporative force), which induces the processes of evaporation, condensation of water vapor in the atmosphere and precipitation. The same force brings about large-scale wind circulation based on the spatial non-uniformity of the magnitude of evaporation flux. Globally averaged

magnitudes of evaporation and precipitation are limited by the flux of solar radiation absorbed by the surface and do not depend on global mean surface temperature.

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## **APPENDIX. Condensational theory of hurricanes and tornadoes**

### **A1. Introduction**

A major physical peculiarity of hurricanes and tornadoes is the large power developed in these wind structures per unit surface area. This power considerably exceeds the global mean flux of solar radiation absorbed by the Earth's surface (Samsury, Zipser, 1995; Wurman et al., 1996) and can therefore be developed only after solar energy has been accumulating for a long time in the form of some kind of potential energy capable of rapid conversion into kinetic energy of air motion. The rate at which locally accumulated potential energy is spent in the area occupied by hurricane or tornado is much higher than the rate at which it has been locally accumulated. The physical nature of potential energy which accumulates slowly and then is rapidly released as a burst of kinetic power has so far remained unclear.

Based on the energy conservation law, the process of energy transformation in the hurricane (or tornado) can be described as follows. Solar energy is stored in the form of some kind of potential energy over a large 'donor' area that greatly exceeds the 'acceptor' area actually occupied by the hurricane at any given moment of time. Then this potential energy is collected from the donor area and concentrated within the area occupied by the hurricane. Energy concentration can occur in space, in time, or both. In the case of spatial concentration of potential energy, the hurricane area does not move, while potential energy is transported from the entire donor area to the hurricane. When energy is concentrated in time, this means that potential energy is not transported elsewhere, but is instantaneously spent on kinetic energy generation in the same local area where it has been accumulated at a rate greatly exceeding the rate of its accumulation. Then it is the hurricane (tornado) itself that must move. The developed wind speeds developed within the hurricane (tornado) can then be sustained if the wind structure as a whole moves at the same speed to the neighboring local area, where it also spends all the accumulated energy, moves further on and so forth. (The same temporal principle of energy concentration is used by locomotive animals as they transect their home range). According to observations, there are no immobile hurricanes, but the velocity of their

movement is always less than the maximum wind velocity within these wind structures. This indicates that hurricanes use both types of energy concentration, temporal and spatial. Tornadoes, which move at much higher speeds than hurricanes, apparently rely predominantly on the second (temporal) principle of potential energy concentration.

Latent heat associated with condensation of water vapor has been so far considered as the single candidate for the store of potential energy that feeds the hurricanes. However, there are no thermodynamic mechanisms transforming latent heat into kinetic energy of air masses. Latent heat is released only after air temperature has decreased due to some independent physical processes; the release of latent heat can only diminish the rate of the temperature drop. In the absence of such processes, no release of latent heat may occur.

Instead, the dynamic fluxes of air are induced by water vapor condensation *per se*, a process in several ways analogous to the phenomenon of osmosis. In the course of condensation water vapor disappears from the gas phase; in the result, local air pressure drops leading to appearance of the wind-inducing pressure gradient. Although latent heat is released during condensation, the condensation-induced drop of pressure represents a different physical process, which is practically independent of the magnitude of the latent heat of vaporization. The volume-specific store of potential energy responsible for hurricane formation is partial pressure  $p_{\text{H}_2\text{O}}$  of saturated water vapor. Condensation of water vapor in hurricanes and tornadoes occurs thousands of times faster than its accumulation in the atmospheric column due to evaporation of moisture from the oceanic surface at the expense of the absorbed solar radiation. Vertical distribution of water vapor partial pressure  $p_{\text{H}_2\text{O}}$  departs significantly from the aerostatic equilibrium; at any height  $p_{\text{H}_2\text{O}}$  is over five times larger than the weight of water vapor column above this height (Makarieva, Gorshkov, 2007), see also Section A2 below. For this reason practically all water vapor ascending in the hurricanes undergoes condensation, so the condensational potential energy coincides with  $p_{\text{H}_2\text{O}}$  to a good approximation.

Latent heat is absorbed while moisture evaporates under the action of solar radiation. Condensation of water vapor leads to the appearance of air pressure gradients and dynamic air flows. The release of latent heat during water vapor condensation, as well as propagation of the air flows occur adiabatically, without the atmosphere exchanging heat with some external medium (hydrosphere or cosmic space). Solar energy acts as the generator

of latent heat  $N_{\text{H}_2\text{O}}Q_{\text{H}_2\text{O}}$  (dimension  $\text{J m}^{-3}$ ,  $N_{\text{H}_2\text{O}}$  and  $Q_{\text{H}_2\text{O}} = 44 \text{ kJ mol}^{-1}$  are the molar density of water vapor and molar heat of vaporization, respectively). Release of the accumulated latent heat during condensation of water vapor serves to transport the potential energy of dynamic air motion. This potential energy has the form of the partial pressure of water vapor  $p_{\text{H}_2\text{O}} \equiv \gamma_{\text{H}_2\text{O}} p = N_{\text{H}_2\text{O}} RT$  (here  $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$  is the universal gas constant,  $T$  is absolute temperature). At  $15 \text{ }^\circ\text{C}$  for the ratio  $N_{\text{H}_2\text{O}}Q_{\text{H}_2\text{O}}/(N_{\text{H}_2\text{O}}RT)$  we have  $N_{\text{H}_2\text{O}}Q_{\text{H}_2\text{O}}/(N_{\text{H}_2\text{O}}RT) = Q_{\text{H}_2\text{O}}/RT = 18$ . The part of solar energy flux that is spent on evaporation is  $N_{\text{H}_2\text{O}}Q_{\text{H}_2\text{O}}\bar{w} \leq 100 \text{ W m}^{-2}$ , where  $\bar{w} \sim 10^{-3} \text{ m s}^{-1}$  is the global mean velocity of the ascending transport of water vapor (Makarieva, Gorshkov, 2007). Within hurricane or tornadoes the ascending transport flux of energy is equal to  $N_{\text{H}_2\text{O}}Q_{\text{H}_2\text{O}}w \geq 10^5 \text{ W m}^{-2}$ , where  $w \geq 1 \text{ m s}^{-1}$  is velocity of vertical wind (in tornadoes  $w \geq 50 \text{ m s}^{-1}$ ); this transport energy flux is thousands of times larger than the primary flux of solar energy. The ascending flux of potential dynamic energy of hurricanes and tornadoes, which is equal to  $p_{\text{H}_2\text{O}}w$ , is 18 times smaller than the flux of transport power. For this reason comparison of this potential energy flux with the flux of solar radiation (see, e.g., Businger, Businger, 2001) is not relevant to the power budget of the hurricane, as the latter is limited by the much larger transport (i.e. latent heat) flux.

Potential energy  $p_{\text{H}_2\text{O}}$  ( $\text{J m}^{-3}$ ) is transformed to kinetic energy  $\rho u_{\text{max}}^2 / 2$  ( $\text{J m}^{-3}$ ) of air masses having density  $\rho$  and moving at velocity  $u_{\text{max}}$  as  $p_{\text{H}_2\text{O}} = \rho u_{\text{max}}^2 / 2$ . At  $\gamma_{\text{H}_2\text{O}} \equiv p_{\text{H}_2\text{O}} / p = 0.02$  at  $15 \text{ }^\circ\text{C}$  or  $\gamma_{\text{H}_2\text{O}} = 0.05$  (at  $30 \text{ }^\circ\text{C}$ ), moist air pressure  $p = 10^5 \text{ Pa}$  and  $\rho = 1.2 \text{ kg m}^{-3}$  we have  $u_{\text{max}} = 50 \text{ m s}^{-1}$  or  $u_{\text{max}} = 90 \text{ m s}^{-1}$ , respectively. These values agree with observations for hurricanes and tornadoes (Samsury, Zipser, 1995; Wurman et al., 1996).

Hurricanes and tornadoes can be compared to an explosion reversed and prolonged in time. In the ordinary explosion potential energy concentrated in the explosion center is released in a burst, making local air pressure rise sharply and causing dynamic air movement in the direction away from the explosion center. Conversely, condensation of saturated water vapor within the column of ascending air in hurricanes and tornadoes leads to a sharp drop of local air pressure. This further enhances the ascending motion of yet accelerating air masses, as well as the compensating radial fluxes of moist air incoming to the area where the process of condensation is most intensive. Water vapor contained in the



incoming air undergoes condensation in the same area; this sustains the pressure difference between the hurricane center and its environment. Hurricane could also be compared to a black hole, which sucks the surrounding air into the center, where it partially “annihilates” due to condensation of water vapor and its disappearance from the gas phase. Thus, hurricane is an “anti-explosion”. While in explosion the gas phase appears from either liquid or solid phase, in hurricanes and tornadoes, conversely, the gas phase of water vapor partially disappears from air due to condensation.

Unlike in explosion, the velocity of air masses in hurricanes and tornadoes is significantly lower than the velocity of thermal molecular motion (sound velocity). In consequence, thermodynamic equilibrium sets in all air volumes, so that air pressure, temperature and density within the hurricane conform to equilibrium thermodynamics. The driving force of all hurricane processes is the release, as in compressed spring, of potential energy accumulated in the form of saturated water vapor in the atmospheric column. The hurricane does not exchange heat with the environment, so all hurricane processes can be described as adiabatic.

Hurricane is not a thermodynamic heat engine based on Carnot cycle (which includes heat consumption during isothermic expansion and heat removal during isothermic compression at different temperatures) or its any modifications where the irreversibility of hurricane processes would be taken into account. Hurricane power and maximum hurricane velocity cannot be quantified on the basis of equilibrium thermodynamics alone. In Section A4 theoretical approaches considering hurricanes as a thermodynamic engine are analyzed in detail; it is shown that such a consideration is physically incorrect.

## A2. Theory of hurricanes

Consider the Bernoulli equation along the streamline (Landau, Lifshitz, 1987)

$$\rho \frac{\partial \mathbf{u}^2}{\partial \mathbf{l}} + \frac{\partial p}{\partial \mathbf{l}} + \rho \mathbf{g} \frac{\partial z}{\partial \mathbf{l}} + \rho \frac{\partial}{\partial \mathbf{l}} \left( C_D \frac{\mathbf{u}^2}{2} \right) = 0, \quad (\text{A1})$$

where  $\rho$  is air mass density,  $p$  is air pressure,  $\mathbf{l}$  is the streamline vector,  $\mathbf{u}$  is wind velocity vector in the hurricane area. The last term describes the forces of turbulent friction acting when the air masses move along the surface. In the hurricane this term is negligibly small,  $C_D u^2 = u_*^2 \ll u^2$ , see (Businger, Businger, 2001) and the explanation below. The very fact of hurricane

existence, i.e. of the large magnitude of wind velocity  $u$ , means that the difference between the pressure gradient force and turbulent friction forces is of the order of the pressure gradient force itself. One can therefore safely neglect turbulent friction in the estimates of hurricane wind velocity and put  $C_D = 0$ , as is done below. Hurricane wind velocities do not exceed one tenth of molecular thermal velocities, so when the main flow decays into smaller eddies, the character of temperature and pressure changes remains adiabatic (with an account of water vapor condensation). For heat increment  $dQ$  we thus have  $dQ = TdS = 0$ , see Section A3 below. The enthalpy increment  $dW = TdS + Vdp = Vdp$ , which enters the Bernoulli equation (Landau, Lifshitz, 1987), is therefore accounted for in (A1).

Integrating (A1) along the horizontal (parallel to axis  $x$ ) and ascending (parallel to the vertical axis  $z$ ) components of the trajectory of air movement we obtain

$$\rho \frac{1}{2} \mathbf{u}^2 - \Delta p = 0, \quad (\text{A2})$$

$$\Delta p = \int_0^{\infty} f_E(z) dz > 0, \quad f_E(z) \equiv -\frac{dp}{dz} - \rho g. \quad (\text{A3})$$

Pressure difference  $\Delta p$  is determined by the fact that water vapor condenses during its ascent.

Let  $l$  and  $h$  be the horizontal and vertical linear dimensions of the hurricane (tornado), respectively. Air flows into the volume occupied by the hurricane (tornado) with velocity  $u_x$  via vertical surface with area of the order of  $lh$  and leaves the hurricane volume flowing in the upward direction via horizontal surface of area  $l^2$  at velocity  $u_z$ . Numerical coefficients at these areas are of the order of unity if one takes into consideration that hurricanes (tornadoes) have the geometry of secularized hemispheres. The continuity equation (law of matter conservation) corresponds then to equality  $u_x h \approx u_z l$ , i.e.  $u_x \approx u_z l/h$ . Smallness of atmospheric height  $h$  in comparison to the horizontal dimensions  $l$  of the hurricane area,  $h \sim 0.01l$ , effectively turns the hurricane into a two-dimensional horizontal surface, where the whole magnitude of pressure difference  $\Delta p$  produced by condensation of water vapor is distributed along the horizontal dimension of the hurricane. For this reason  $\Delta p = p_a - p_c$ , where  $p_a$  and  $p_c$  are surface air pressures outside the hurricane and in the hurricane center, respectively. This result follows from (A2) and the continuity equation according to which  $u_x = u_z l/h \gg u_z$ , and, consequently,  $u^2 = u_x^2 + u_z^2 = u_x^2(1 + h^2/l^2) \approx u_x^2$  for hurricanes at  $h/l \sim 0.01$ . For tornadoes that, conversely, have  $h/l > 1$  and,

consequently,  $u_z > u_x$ , the major pressure difference related to water vapor condensation falls on the horizontal dimension  $h$ .

Hurricane power per unit area of the vertical cross-section perpendicular to horizontal wind velocity is  $\rho u^3/2 = \Delta p u$ . Power of turbulent dissipation of hurricane energy at the surface is, per unit surface area, equal to  $\rho(u^2/2)u_z = \rho(u_*^2/2)u \equiv C_D \rho u^3/2$ . It follows that  $C_D \approx u_z/u \ll 1$ , so power losses to turbulent friction at the surface are negligibly small compared to hurricane's power. Hurricanes and tornadoes accumulate kinetic energy of wind until the entire store  $\Delta p$  of potential energy is converted to kinetic energy. Further growth of hurricane wind speeds is impossible even in the absence of turbulent friction forces. From (A2) we have for hurricanes

$$\rho \frac{u_x^2}{2} = \Delta p, \quad \Delta p = \int_0^\infty f_E(z) dz, \quad f_E \equiv -\frac{\partial p}{\partial z} - \rho g, \quad u_z = u_x \frac{h}{l}. \quad (\text{A4})$$

Let us now determine force  $f_E(z)$  of moist air. Pressure  $p$  of moist air is equal to the cumulative partial pressure  $p_d$  of dry air constituents plus partial pressure  $p_{\text{H}_2\text{O}}$  of water vapor. We will consider  $p_{\text{H}_2\text{O}}$  as saturated pressure, because the area where water vapor is not saturated and where no condensation occurs does not contribute to the integral for  $\Delta p$  in (A4). Taking these considerations into account we have

$$f_E(z) = f_d(z) + f_{\text{H}_2\text{O}}, \quad f_d = -\frac{\partial p_d}{\partial z} - \rho_d g, \quad f_{\text{H}_2\text{O}}(z) = -\frac{\partial p_{\text{H}_2\text{O}}}{\partial z} - \rho_{\text{H}_2\text{O}} g. \quad (\text{A5})$$

The dry component of atmospheric air is in hydrostatic equilibrium due to the observed independence of molar mass  $M_d$  of dry air on height  $z$ , so  $f_d = 0$  (Makarieva, Gorshkov, 2007) and  $f_E(z) = f_{\text{H}_2\text{O}}(z)$ . The value of  $f_E$  can now be found invoking Clausius-Clapeyron equation:

$$dp_{\text{H}_2\text{O}} = p_{\text{H}_2\text{O}} \frac{Q_{\text{H}_2\text{O}}}{RT^2} dT \quad \text{or} \quad -\frac{\partial p_{\text{H}_2\text{O}}}{\partial z} = p_{\text{H}_2\text{O}} \frac{Q_{\text{H}_2\text{O}} \Gamma}{RT^2}, \quad \Gamma \equiv -\frac{\partial T}{\partial z}. \quad (\text{A6})$$

With use of (A6) we have

$$-\frac{\partial p_{\text{H}_2\text{O}}}{\partial z} = \frac{p_{\text{H}_2\text{O}}}{h_{\text{H}_2\text{O}}}, \quad h_{\text{H}_2\text{O}} \equiv \frac{T^2 R}{\Gamma Q_{\text{H}_2\text{O}}}, \quad (\text{A7})$$

where  $h_{\text{H}_2\text{O}}$  is the scale height of the exponential decline of the saturated pressure of water vapor at a given value of the vertical lapse rate  $\Gamma$  (A6) of air temperature.

Using the ideal gas equation of state

$$p = \rho gh, \quad h \equiv \frac{RT}{Mg} = 8.4 \text{ km}, \quad p_{\text{H}_2\text{O}} = \rho_{\text{H}_2\text{O}} gh_v, \\ h_v = \frac{RT}{M_v g} = 13.5 \text{ km}, \quad \frac{M_v}{M} = 0.62, \quad (\text{A8})$$

we arrive at the following expression for force  $f_E$ :

$$f_E = \frac{p_{\text{H}_2\text{O}}}{h_{\text{H}_2\text{O}}}(1 - \alpha), \quad \alpha \equiv \frac{h_{\text{H}_2\text{O}}}{h_v} \approx 0.17, \quad p_{\text{H}_2\text{O}} = \rho gh \gamma_{\text{H}_2\text{O}}, \quad \gamma_{\text{H}_2\text{O}} \equiv \frac{p_{\text{H}_2\text{O}}}{p}, \quad (\text{A9})$$

Here scale heights  $h$  and  $h_v$  define how pressures of air and water vapor, respectively, exponentially diminish with height in the state of aerostatic equilibrium. The value of  $\alpha$  in (A9), which at a given height is equal to the ratio between the weight of the water vapor column above that height to local pressure of water vapor in (A5), depends on  $\Gamma \equiv -\partial T / \partial z$ , see (A7). It is small at the observed global mean value of  $\Gamma = \Gamma_{ob} = 6.5 \text{ K km}^{-1}$ , which corresponds to  $h_{\text{H}_2\text{O}} \approx 2.4 \text{ km}$ . Aerostatic equilibrium of saturated water vapor, which satisfies the condition  $h_{\text{H}_2\text{O}} = h_v$ ,  $\alpha = 1$  and  $f_E = 0$ , arises, according to (A7), at  $\Gamma = T/H$ , where  $H \equiv M_v g / Q_{\text{H}_2\text{O}} = 250 \text{ km}$ . Solving the latter equation for  $\Gamma$  yields  $\Gamma = \Gamma_{\text{H}_2\text{O}} \exp(-z/H) \approx \Gamma_{\text{H}_2\text{O}} = TM_v g / Q_{\text{H}_2\text{O}} = 1.2 \text{ K km}^{-1} < \Gamma_{ob} = 6.5 \text{ K km}^{-1}$ .

In hurricanes it is natural to term force  $f_E$  as the condensational force. Stores of water vapor, which condensation determines the hurricane power, have been accumulating for a long period of time preceding the hurricane development. As discussed above, such accumulation occurs at a power equal, in its order of magnitude, to that of solar radiation absorbed by the surface. This power is hundreds of times smaller than the power at which water vapor is “spent” (condensed) within the hurricane. Using definitions (A7) and (A8) condensational force  $f_E$  can be written as

$$f_E = \rho \frac{u_{\text{max}}^2}{2} \frac{1}{h_{\text{H}_2\text{O}}}, \quad \alpha \equiv h_{\text{H}_2\text{O}} / h_v = \Gamma_{\text{H}_2\text{O}} / \Gamma_{ob}, \\ u_{\text{max}} \equiv [(2p\gamma_{\text{H}_2\text{O}} / \rho)(1 - \alpha)]^{1/2} = [(2gh\gamma_{\text{H}_2\text{O}} / \rho)(1 - \alpha)]^{1/2}. \quad (\text{A10})$$

Work of this force is

$$\int_0^{\infty} f_E(z) dz \approx f_E(0) h_{\text{H}_2\text{O}} \approx gh_s \gamma_{\text{H}_2\text{O}s} (1 - \alpha) \approx \rho_s \frac{u_{\text{max}}^2}{2}, \quad (\text{A11})$$

where low index  $s$  corresponds to the planetary surface at  $z = 0$ ,  $\gamma_{\text{H}_2\text{O}}$  (A9) is the volume ratio of the saturated water vapor in moist air. Expression (A11) describes the potential energy of condensation. This energy, with an account of turbulent friction force, determines wind velocity  $u$  and pressure drop  $\Delta p$  in the hurricane, see (A4):

$$\Delta p = p_s \gamma_{\text{H}_2\text{O}s} (1 - \alpha) = \rho_s \frac{u_{\text{max}}^2}{2} = \rho_s \frac{u^2}{2}. \quad (\text{A12})$$

At 30 °C in the tropics we have  $\gamma_{\text{H}_2\text{O}} = 0.05$ ,  $\rho_s = 1.2 \text{ kg m}^{-3}$ ,  $p_s = 10^5 \text{ Pa} = 1 \text{ bar}$ . Using these values we obtain the following estimates of  $u$  and  $\Delta p$  in the hurricane, see (A4) and (A11):

$$\Delta p = 50 \text{ mbar}, \quad u = u_{\text{max}} = 80 \text{ m s}^{-1}. \quad (\text{A13})$$

These theoretical estimates are in satisfactory agreement with observations (Samsury, Zipser, 1995). At air temperature of 40 °C we have  $\gamma_{\text{H}_2\text{O}} = 0.09$ .

This gives  $\Delta p = 90 \text{ mbar}$  and  $u = u_{\text{max}} = 100 \text{ m s}^{-1}$ . Such extreme maximum values of temperature, water vapor partial pressure, pressure drop, and wind velocity are characteristic of tornadoes on land (Wurman et al., 1996).

### A3. Adiabatic thermodynamics of hurricanes

Horizontal movement of air towards the hurricane center and the ascent of air in the area of maximum wind speed under the action of the evaporative force occur without an external heat input. This allows one to use the adiabatic condition to relate changes of air pressure and temperature that take place in the horizontal and vertical air flows. This condition is formulated using the first law of thermodynamics as

$$dQ = TdS = c_p dT + Q_{\text{H}_2\text{O}} d\gamma_{\text{H}_2\text{O}} - V dp, \quad dQ = 0, \quad (\text{A14})$$

where  $T$  is absolute temperature (K),  $\gamma_{\text{H}_2\text{O}} \equiv p_{\text{H}_2\text{O}} / p \equiv V / V_{\text{H}_2\text{O}}$  (dimensionless),  $p$  and  $p_{\text{H}_2\text{O}}$  are pressures ( $\text{J m}^{-3} \equiv \text{N m}^{-2} \equiv \text{Pa}$ ) and  $V$  and  $V_{\text{H}_2\text{O}}$  are molar volumes ( $\text{m}^3 \text{ mol}^{-1}$ ) of moist air and saturated water vapor, respectively;  $Q$  is molar heat ( $\text{J mol}^{-1}$ ),  $c_p$  and  $S$  are molar specific heat of moist air at constant pressure and entropy, respectively ( $\text{J mol}^{-1} \text{ K}^{-1}$ ),  $Q_{\text{H}_2\text{O}} = 44 \text{ kJ mol}^{-1}$  is molar heat of water evaporation, which can be to a good approximation put constant (temperature-independent). Equation (A14) is written in terms of molar quantities that are conventional in physics. Mass-specific quantities, which one commonly operates with in meteorology, can be obtained from these by dividing the corresponding molar terms by air molar mass  $M = 29 \text{ g mol}^{-1}$ .

The second term in (A14) describes the amount of heat that is released due to vapor condensation or absorbed due to water evaporation. Partial pressure of water vapor changes due to two physical processes. One is the change proportional to the change of total air pressure (i.e. the non-condensable air constituents) and the other is the change due to possible phase transitions (evaporation or condensation of moisture). It is only the second change that is accompanied by the absorption or release of latent heat. Total relative change of water vapor partial pressure is  $d(\gamma_{\text{H}_2\text{O}}p)/p$ . From this we subtract the change related to the change of air pressure,  $\gamma_{\text{H}_2\text{O}}dp/p$ ; multiplying the resulting difference by  $Q_{\text{H}_2\text{O}}$  we obtain that the heat released or absorbed during the phase transition is equal to  $Q_{\text{H}_2\text{O}}[d(\gamma_{\text{H}_2\text{O}}p - \gamma_{\text{H}_2\text{O}}dp)]/p = Q_{\text{H}_2\text{O}}d\gamma_{\text{H}_2\text{O}}$ , i.e. to the second term in (A14).

Interaction of the atmosphere with the Earth's surface takes the form of matter and energy flows determined by processes of molecular diffusion and heat conductivity in the thin near-surface layer of thickness  $\sim 50 \mu\text{m}$ . Power of the non-radiative fluxes of sensible and latent heat from land or oceanic surface to the atmosphere is limited by the power of the absorbed flux of solar radiation, which is of the order of  $100 \text{ W m}^{-2}$ . As estimated in Section A1, power of the vertical flux of latent heat release in hurricanes exceeds solar power by thousands of times. This means that flux of heat input from the surface represents a negligible term in the energy budget of the hurricane. All atmospheric processes of the hurricane are strictly adiabatic and conform to the adiabatic condition (A14).

Invoking Clausius-Clapeyron equation (A6) to relate  $dp_{\text{H}_2\text{O}}$  and  $dT$  one can re-write Eq. (A14) as

$$dQ = TdS = c_p dT \left[ 1 + \frac{R}{c_p} \gamma_{\text{H}_2\text{O}} \left( \frac{Q_{\text{H}_2\text{O}}}{RT} \right)^2 \right] - RT \frac{dp}{p} \left( 1 + \gamma_{\text{H}_2\text{O}} \frac{Q_{\text{H}_2\text{O}}}{RT} \right). \quad (\text{A15})$$

The hurricane does not exchange heat with its environment,  $dQ = 0$ , which makes it possible to relate relative changes of temperature and pressure with use of (A15) as

$$\frac{dT}{T} = \frac{dp}{p} \frac{R}{c_p} \varphi(T, \gamma_{\text{H}_2\text{O}}), \quad \varphi(T, \gamma_{\text{H}_2\text{O}}) \equiv \frac{1 + \gamma_{\text{H}_2\text{O}} \frac{Q_{\text{H}_2\text{O}}}{RT}}{1 + \frac{R}{c_p} \gamma_{\text{H}_2\text{O}} \left( \frac{Q_{\text{H}_2\text{O}}}{RT} \right)^2}, \quad (\text{A16})$$

$$\frac{R}{c_p} = 0.29; \quad \varphi(300 \text{ K}, 0.04) = 0.37; \quad \varphi(300 \text{ K}, 0.04) \frac{R}{c_p} = 0.11. \quad (\text{A17})$$

According to (A16)-(A17), in all moist adiabatic processes drop of pressure is accompanied by decrease of temperature and precipitation of moisture, with the relative change of temperature being approximately one tenth of the corresponding relative change of pressure. Note that expression for  $\varphi(T, \gamma_{\text{H}_2\text{O}})$  is written in terms of molar-specific, not mass-specific, quantities.

Since in the hurricane  $dQ = 0$  both along the horizontal path of air masses along  $x$  axis, as well as along the vertical path of their ascent along  $z$  axis, we have for the corresponding changes of temperature and pressure:

$$\frac{1}{T} \frac{\partial T}{\partial x} = \frac{1}{p} \frac{\partial p}{\partial x} \frac{R}{c_p} \varphi(T, \gamma_{\text{H}_2\text{O}}), \quad (\text{A18})$$

$$\Gamma_m \equiv -\frac{\partial T}{\partial z} = \frac{Mg}{c_p} \varphi(T, \gamma_{\text{H}_2\text{O}}), \quad -\frac{1}{p} \frac{\partial p}{\partial z} = \frac{Mg}{RT}, \quad (\text{A19})$$

where  $M = 29 \text{ g mol}^{-1}$  is air molar mass. Numeric values in (A17) were obtained putting  $c_p = c_{pd} = 29 \text{ J mol}^{-1} \text{ K}^{-1}$ . Specific heat of water vapor at constant pressure is  $32 \text{ J mol}^{-1} \text{ K}^{-1}$ , i.e. it deviates from  $c_{pd}$  (specific heat of dry air at constant pressure) by 10%. Given the low mixing ratio of water vapor in moist air, the difference between  $c_p$  and  $c_{pd}$  is negligibly small.

Expression for  $\Gamma_m$  (A19) is known in meteorology as the moist-adiabatic lapse rate of air temperature. This expression is written in the approximation, see Section A2,

$$\begin{aligned} \frac{1}{p} \frac{\partial p}{\partial z} &= \frac{1}{p_d + p_{\text{H}_2\text{O}}} \left( \frac{\partial p_d}{\partial z} + \frac{\partial p_{\text{H}_2\text{O}}}{\partial z} \right) = \frac{1}{p_d(1 + \gamma_{\text{H}_2\text{O}})} \left( \frac{p_d}{h_d} + \frac{p_{\text{H}_2\text{O}}}{h_{\text{H}_2\text{O}}} \right) = \\ &= \frac{1}{h_d} \Psi(T, \gamma_{\text{H}_2\text{O}}), \quad \Psi(T, \gamma_{\text{H}_2\text{O}}) \equiv \frac{1 + \frac{\gamma_{\text{H}_2\text{O}}}{\beta}}{1 + \gamma_{\text{H}_2\text{O}}}, \quad \beta \equiv \frac{h}{h_{\text{H}_2\text{O}}} \geq 3.5, \quad (\text{A20}) \end{aligned}$$

where  $\Psi(T, \gamma_{\text{H}_2\text{O}}) \approx 1 - \gamma_{\text{H}_2\text{O}} \left( 1 - \frac{1}{\beta} \right)$  is replaced by unity. As far as  $\gamma_{\text{H}_2\text{O}} \leq 0.04$ , this approximation, which corresponds to hydrostatic equilibrium of moist air, appears to be valid for the calculation of  $\Gamma_m$  to the accuracy of the neglected terms, which do not exceed 4%, i.e. along the vertical we have  $\Delta_z p = p \gg p_{\text{H}_2\text{O}}$  at  $\Delta z = h_d$ . This fact might have served as the main

justification for the incorrect statement that hydrostatic equilibrium of moist air is a good approximation not just for the calculation of  $\Gamma_m$ , but also for all other major problems of theoretical meteorology. However, in the horizontal dimension the application of hydrostatic equilibrium,  $f_E = 0$  in (A4), results in the main term in (A14) being lost, the one determining horizontal wind velocity,  $\Delta p \approx p_{\text{H}_2\text{O}}$ , see also discussion of Eq. (A21) below.

Using (A16), (A19) it is possible to estimate the moist-adiabatic vertical lapse rate that arises in the ascending air masses in the atmospheric column of the hurricane. The value of  $\varphi(T, \gamma_{\text{H}_2\text{O}}) \approx \varphi(300 \text{ K}, 0.04) = 0.37$  remains practically unaffected by the decrease of air temperature up to the heights of the order of  $h_{\text{H}_2\text{O}} \sim 3 \text{ km}$  that are of interest. This yields  $\Gamma_m = 4.7 \text{ K km}^{-1}$ , which is four times the critical value of air temperature lapse rate  $\Gamma_{\text{H}_2\text{O}} = 1.2 \text{ K km}^{-1}$ , below which the non-equilibrium character of the vertical distribution of atmospheric vapor no longer holds and force  $f_E$  (A9) disappears. This is the main conclusion for the hurricane energetics that can be drawn from the analysis of the equilibrium thermodynamics of the ideal gas of the atmosphere.

Note that the horizontal adiabatic movement of air masses towards the hurricane center is also accompanied by a drop of temperature, condensation of moisture and, in consequence, by the appearance of the horizontal condensation force directed inward to the hurricane center. However, according to (A18) and (A19), the horizontal temperature gradient and the associated horizontal condensation force are hundreds of times smaller than the corresponding vertical magnitudes and as such can be neglected. Additionally, surface air near the hurricane center cools due to precipitation of moisture condensed at heights  $z \sim h_{\text{H}_2\text{O}}$  at temperatures lower than the surface temperature by about 10 K. However, the relative contribution of this effect into hurricane's energetics is small as well. It is of the order of the ratio of (specific heat of liquid water multiplied by the temperature difference) to latent heat of vaporization, thus not exceeding several per cent of the main physical effect taken into account.

Release of latent heat due to drop of air temperature during the adiabatic ascent of air masses reduces, in accordance with the Le-Chatellier principle, the adiabatic temperature lapse rate from its maximum value which would be observed in dry air with  $\gamma_{\text{H}_2\text{O}} = 0$  и  $\varphi(T, \gamma_{\text{H}_2\text{O}}) = 1$ . This heat cannot be transformed into the dynamic energy of air movement; such



a process would violate the second law of thermodynamics. It should be emphasized that the adiabatic cooling accompanied by release of latent heat is the consequence, not cause, of the ascending air movement, the latter invoked by condensation force  $f_E$ . It is the whole atmospheric column that ascends rather than particular air volumes overheated compared to their surroundings. Not the Archimedes force, but condensation force  $f_E$ , which equally acts on air volumes with neutral or occasionally positive or negative buoyancy makes air masses rise. This explains the observation that the buoyancy of vertical motions in hurricane cores can be either negative or positive (Eastin et al., 2005).

#### **A4. Analysis of the theoretical accounts of hurricanes**

We will now analyze the major physical problems faced by the existing theoretical accounts of hurricanes. We start with a list of general statements.

1) The assumption that the exchange of heat with the ocean is the main contributor to the hurricane's power is incorrect. Thermal energy can only leave the hurricane area in the form of thermal radiation into space. All other types of energy are taken into account in the adiabatic equation (A14). The vertical flux of latent heat released in the ascending air masses within the hurricane (Section A3, formulae (A16), (A18)), which transports the dynamic power of the hurricane (Section A1), is thousands of times greater than the flux of the absorbed solar radiation. If this flux had been converted to thermal power in the area occupied by the hurricane, thermal radiation to space from this area would have been thousands of times greater than the global mean flux of thermal radiation into space, the latter corresponding to brightness temperature  $T_b = 255$  K that is by 33 K lower than the global mean surface temperature  $T_s = 288$  K. This is physically prohibited: as far as, according to Stephan-Boltzmann law, the flux of thermal radiation is proportional to the fourth power of brightness temperature, in such a thermal radiation from areas occupied by hurricanes would have had a temperature in excess of  $T_b(1000)^{1/4} \sim 1400$  K. Consequently, neither the power of latent heat release, nor the dynamic power of the hurricane can be maintained at the expense of the on-going heat absorption from some external medium, e.g., from the ocean. This would violate the second law of thermodynamics: in the absence of heat sink, thermal energy of the ocean cannot be converted to mechanical work. In reality, hurricane's energy is transported far away from the hurricane area in the form of released latent heat and of the kinetic energy of small eddies. It further dissipates to thermal radiation and is emitted to space from a large area at a power

similar in its order of magnitude to the global mean power of the absorbed solar radiation.

2) Work of turbulent friction forces turns the dynamic energy of the large-scale air flow into dynamic energy of the smaller eddies. This work is not equal to the thermodynamic heat, as is assumed in a number of theoretical studies. During such work, while the larger eddies decay into smaller ones, all thermodynamic parameters change in the adiabatic regime, i.e. without absorption or emission of heat to the environment at constant entropy, in agreement with all the available observations.

3) Hurricane wind velocities  $u$  are dictated by the observed pressure difference  $\Delta p$  between the hurricane and the outer environment. In the thermodynamic consideration of hurricanes the nature of this difference remains unexplained. On the basis of thermodynamics, it is not possible to obtain an expression for the pressure difference or hurricane wind velocity neglecting the fact that, due to condensation of moisture in the atmosphere, vertical distribution of water vapor deviates from the aerostatic equilibrium, which leads to the appearance of force  $f_E$  (A9).

For a detailed analysis of the theory of hurricanes presented in the works of Emmanuel (1986, 1991, 1995, 2003, 2005, 2006) and Bister and Emmanuel (1998) (below these papers are referred to as E1986, E1991, E1995, E2003, E2005, E2006) and BE1998) we will now quantify all the terms of (A14) written for horizontal air flow using the observed value of the maximum drop of pressure  $\Delta p/p \sim 0.1$  in hurricanes;  $\Delta p = \Delta_x p = p_a - p_c > 0$ , where  $p_a$  and  $p_c$  stand for atmospheric pressure outside the hurricane and in the hurricane's center, respectively. Taking into account the adiabatic nature of all processes,  $\Delta Q = 0$ , we have

$$c_p \Delta_x T + Q_{\text{H}_2\text{O}} \Delta_x \gamma_{\text{H}_2\text{O}} - V \Delta p = 0 \quad (\text{A21})$$

$$\text{J mol}^{-1} \quad 40 \quad + \quad 80 \quad - \quad 120 = 0$$

For the sake of clarity, the numerical values in (A21) are rounded within the 10% accuracy. All the terms of (A21) are of one and the same order of magnitude. The largest term is the third (gradient) term  $V \Delta p = M \Delta p / \rho$ , which determines the magnitude of hurricane wind velocity  $u$  in (A2). The first term, which is related to the decrease of temperature, is the smallest one at one third of the largest term  $M \Delta p / \rho$ . This term is essential, however, as far as condensation of saturated water vapor at the surface occurs namely due to this drop of temperature. This leads to the change of  $\gamma_{\text{H}_2\text{O}}$  and the appearance of the second term in (A21), which makes up two thirds of the largest, third term. One can easily calculate the magnitude by

which air temperature at the surface drops as air moves from outside the hurricane towards the hurricane center. From (A16) and (A17) we have  $\Delta_x T/T = (\Delta p/p) \times (R/c_p) \times \varphi(T, \gamma_{\text{H}_2\text{O}}) = 0.11 \times (\Delta p/p)$ . Thus, for the majority of hurricanes with  $\Delta p/p \sim 0.05$  temperature difference between the hurricane center and outer environment does not exceed  $\Delta_x T \sim 1.6$  K. This agrees with observations, see, e.g., Fig. 12 of Black and Holland (1995).

To obtain mass-specific magnitudes ( $\text{J kg}^{-1}$ ) instead of molar ones ( $\text{J mol}^{-1}$ ) each term in (A21) must be divided by air molar mass  $M = 29 \text{ g mol}^{-1}$ . To obtain volume-specific magnitudes ( $\text{J m}^{-3}$ ) every term in (A21) has to be multiplied by molar density  $N \equiv V^{-1}$  ( $\text{mol m}^{-3}$ ). To estimate horizontal fluxes of these terms per unit area of hurricane's vertical cross-section ( $\text{W m}^{-2}$ ) (their cumulative flux is equal to zero), it is necessary to multiply each term of (A21) by  $Nu_{\text{max}} \equiv \rho u_{\text{max}}/M$  ( $\text{mol m}^{-2} \text{ s}^{-1}$ ).

To estimate vertical fluxes of these terms per unit surface area ( $\text{W m}^{-2}$ ) (total flux is zero), it is necessary to multiply each term of (A21) by  $Nu_z \equiv \rho(h_{\text{H}_2\text{O}}/l)u_{\text{max}}/M$  ( $\text{mol m}^{-2} \text{ s}^{-1}$ ) ( $h_{\text{H}_2\text{O}} \ll l$ ). Such vertical fluxes will be of the order of  $\rho u_{\text{max}}^2 u_z \sim 10^3 \text{ W m}^{-2}$ , see, e.g., Table 1 of Black and Holland (1995). It should be emphasized that these fluxes are not related to the fluxes of latent, sensible or radiative heat from the (oceanic) surface that are of the order of  $100 \text{ W m}^{-2}$ . As far as  $\Delta_x T \ll \Delta_z T$  and  $\Delta_x \gamma_{\text{H}_2\text{O}} \ll \Delta_z \gamma_{\text{H}_2\text{O}} \sim \gamma_{\text{H}_2\text{O}}$ , they neither characterize the vertical flux of latent heat release in the hurricane, which, as discussed in Section A1, see also explanations to formula (A20), exceeds  $10^5 \text{ W m}^{-2}$ .

According to the first law of thermodynamics (energy conservation law), work  $A$  performed by thermodynamic engines that receive heat  $\Delta Q_s > 0$  at temperature  $T_s$  and lose heat  $\Delta Q_0 > 0$  at temperature  $T_0$ , is equal to

$$\Delta Q_s = \Delta Q_0 + A \quad \text{or} \quad A = \Delta Q_s - \Delta Q_0, \quad T_s > T_0. \quad (\text{A22})$$

The second law of thermodynamics relates  $\Delta Q_s$  and  $\Delta Q_0$  to temperatures  $T_s$  and  $T_0$  in the reverse heat engines as

$$|\Delta S| \equiv \frac{\Delta Q_s}{T_s} = \frac{\Delta Q_0}{T_0}, \quad \text{i.e.,} \quad \Delta Q_0 = \frac{T_0}{T_s} \Delta Q_s. \quad (\text{A23})$$

From the first and second laws of thermodynamics we have:

$$A = \frac{T_s - T_0}{T_s} \Delta Q_s, \quad \varepsilon \equiv \frac{A}{\Delta Q_s} = \frac{T_s - T_0}{T_s} < 1. \quad (\text{A24})$$

The magnitude of  $\varepsilon$ , which is the efficiency of Carnot's reversible heat cycle, determines the maximum possible efficiency at which work can be produced in reversible heat engines. In real engines there are always irreversible heat losses on friction; their efficiency is invariably lower than  $\varepsilon$  (A24). Work produced by heat engines can be converted to potential energy of chemical or gravitational nature, or it can be transformed into practically non-dissipating kinetic energy like, for example, the kinetic energy of satellites rotating around the Earth, and stored in these forms. Or it can dissipate with the release of an amount of heat equal to  $A$ , but this can only occur outside the work-producing heat engine.

The amount of heat released in the course of dissipation of work  $A$  is unrelated to the amount of heat  $\Delta Q_s$  consumed by the heat engine from the heat source. When work  $A$  is identified with  $\Delta Q_s$  or if  $\Delta Q_s$  is interpreted as the sum of  $\Delta Q_s$  and  $A$ , the first (A22) or second (A23), (A24) laws of thermodynamics are violated. This is the main physical inconsistency of the aforementioned theoretical accounts of hurricanes.

Indeed, if  $\Delta Q_s = A$ , as it is assumed in E1986, E1991, E1995, then it follows from (A24) that  $\varepsilon = 1$ . Consequently, either  $\Delta Q_0 = 0$ , or  $T_0 = 0$ . If  $T_0/T_s = 2/3$ , as it is assumed in E1986-2006, then from the second equality in (A23) and the condition  $\Delta Q_0 = 0$  it follows that  $\Delta Q_s = 0$  and  $A = 0$ , i.e. the heat engine does not exist. The observation that hurricanes do exist and produce a non-zero work  $A$  lacks a thermodynamic explanation.

If, on the other hand, one assumes that heat which forms in the course of dissipation of work  $A$  can be added to heat  $\Delta Q_s$ , as it is done in BE1998, E2003, E2005, E2006, then, instead of (A22), we have

$$A = \frac{T_s - T_0}{T_s} (\Delta Q_s + A) \quad \text{or} \quad A = \frac{T_s - T_0}{T_0} \Delta Q_s. \quad (\text{A25})$$

This relationship, explicitly present in BE1998, E2003, E2005, E2006, can be written in the following form

$$A = \Delta Q_s - \Delta Q_0 + \varepsilon A, \quad \varepsilon \equiv \frac{T_s - T_0}{T_s} \approx \frac{2}{3}, \quad (\text{A26})$$

which makes it clear that (A25) explicitly violates the first law of thermodynamics (A22), i.e. the fundamental energy conservation law. Generally, already the appearance of the multiplier  $(T_s - T_0)/T_0$  in the second expression of (A25) is to raise suspect in every physicist: temperature  $T_0$  can be chosen such that  $(T_s - T_0) > T_0$ , so that  $A > \Delta Q_s$ ; besides, at  $T_0 \rightarrow 0$  we have  $A \rightarrow \infty$ .

We will now indicate the particular places and formulae in E1986, E1991, E1995, E2003, E2005, E2006, BE1991 physical errors are made. E1991 reviews preceding studies, so we start with the analysis of this paper, see also (Holland, 1997). Everywhere below numbers of formulae taken from E1991 are preceded by "E".

Formula (E15) of E1991 is, according to E1991, obtained by integrating the Bernoulli equation (E1) along the horizontal streamline  $ac$  from the outer environment  $a$  to hurricane center  $c$ . This formula does not contain velocity  $V^2$ , which is present in the Bernoulli equation (E1) and, as such, corresponds to the equality between work of turbulent friction forces and work of pressure gradient forces  $A$ . Work of turbulent friction forces on the other parts of the trajectory of air masses is considered to be negligible (see p. 185, third paragraph from bottom in E1991), i.e.  $\oint \mathbf{F}dl = \int_a^c \mathbf{F}dl$ .

Already from (E15) it can be concluded that hurricane cannot exist: pressure gradient forces are exactly compensated by turbulent friction forces  $\mathbf{F}$ , only in this case velocity  $V = 0$  can be dropped from (E15). In the result, (E15) can be written as

$$A = \int_a^c \mathbf{F}dl = - \int_a^c \alpha dp = RT_s \ln \frac{p_a}{p_c} \approx \alpha_s \Delta p, \left( \alpha_s \equiv \frac{RT_s}{p_a}, \Delta p = p_a - p_c \right). \quad (\text{E15})$$

Joint consideration of (E15) and the following formulae of E1991

$$\oint T ds = \oint \mathbf{F}dl, \quad (\text{E4})$$

$$\varepsilon T_s \Delta s = \oint \mathbf{F}dl, \quad (\text{E11})$$

in the view of (A22) yields  $\varepsilon = 1$ :

$$\Delta Q_s - \Delta Q_0 = \varepsilon \Delta Q_s = \oint \mathbf{F}dl = \int_a^c \mathbf{F}dl = A, \quad \text{i.e., } \varepsilon = 1$$

$$(\Delta Q_s \equiv T_s \Delta s, \quad \Delta Q_0 \equiv T_0 \Delta s.)$$

At  $\varepsilon = 1$  formulae (E16) and (E7)

$$\varepsilon T_s \Delta s = RT_s \ln \frac{p_a}{p_c}, \quad (\text{E16})$$

(in (E16) the last term is dropped due to its negligibly small magnitude, as estimated in E1991)

$$T_s \Delta s = RT_s \ln \frac{p_a}{p_c} + L_v (q_c - q_a), \quad (\text{E7})$$

where  $q_a$  and  $q_c$  are the mass shares of water vapor in the atmosphere outside the hurricane and in the hurricane center, respectively,  $L_v = Q_{\text{H}_2\text{O}}/M$  is mass-specific heat of vaporization, yield

$$L_v(q_c - q_a) = 0,$$

which means that the flux of latent heat from the ocean to the atmosphere is zero. Taking this result into account and recalling that in E1991 it is assumed that the process along streamline  $ac$  is isothermic,  $\Delta T = 0$ , one obtains from (E15) and (E2)

$$Tds = c_p dT + d(L_v q) - \alpha dp, \quad (\text{E2})$$

that

$$\Delta Q_s \equiv T_s \Delta S = \alpha_s \Delta p = A.$$

To summarize, observed mechanical work  $A$  of the hurricane appears to be equal to heat increment  $\Delta Q_s$ , which does not have a physical meaning (in particular, it is not related to the flux of latent heat, which is zero) and cannot be derived from anywhere. Hurricane's energetics remains unexplained. As shown above, in reality  $\Delta Q_s = 0$  (A21) and work  $A$  is determined by different physical magnitudes.

Starting from E1995, the logic of thermodynamic calculations changes. Heat increment  $\Delta Q_s$  is now related not to the horizontal difference between the atmospheric thermodynamic parameters inside and outside the hurricane, but to the vertical difference between the thermodynamic parameters of air in the hurricane and air in the narrow layer at the air-sea interface. Thickness of this transition layer where all processes are driven by molecular diffusion is about  $50 \mu\text{m}$  above the water surface. The contribution of processes within this layer to hurricane energy budget is of the order of the ratio between thickness of the transition layer and thickness of atmospheric layer  $h_{\text{H}_2\text{O}} \sim 2 \text{ km}$  where water vapor condensation and the hurricane actually take place. This ratio is about  $10^{-8}$ , so the microscopic surface layer makes no impact on hurricane energetics.

In E1995 the huge volume difference between the considered layers is overlooked. Instead, the difference in heat increments between the transition layer and the atmosphere is calculated in terms of mass-specific values, i.e. per unit air mass. This is equivalent to the difference of molar values divided by air molar mass  $M$ . Air pressures in the atmosphere ( $a$ ) and in the surface layer ( $s$ ) being equal, molar (or mass) difference of heat increments in these layers is equal to the difference of their molar (or mass) enthalpies,  $(k_s^* - k_a)$  in the notations of E1995. This difference of molar enthalpies, given the neglected difference in the volumes of the considered layers, is

physically meaningless. But numerically it is of the order of the two first terms in (A21). Since the sum of all the three terms of (A21) is equal to zero, the difference of molar enthalpies coincides, by the order of magnitude, with the absolute value of the third gradient term  $V\Delta p$ , which determines mechanical work  $A$  of the hurricane per mol air. Therefore, E1995, as earlier E1991, makes use of the equality  $\varepsilon\Delta Q_s = \varepsilon(k_s^* - k_a) = A$ , which violates the second law of thermodynamics, as far as  $\Delta Q_0 = 0$  at  $T_0/T_s = 2/3$ , see (A22)-(A24), and, hence,  $\varepsilon = 1$ . The magnitude of  $(k_s^* - k_a)$ , see formula (3) in E1995, is estimated from the observed gradient term  $V\Delta p$  (A21), although the main task of the theory of hurricanes namely consists in the theoretical quantification of this term.

Formula (8) in E2003 is equivalent to (A25) (see also formulae (5)-(7) in E2003, where (8) is multiplied by  $\rho V$ ), which violates the energy conservation law, see (A25), (A26). This relationship is present in all subsequent works starting from BE1998 (see formula (20) and (21) therein), including E2005 and E2006.

Work of BE1998, which aims to quantify the input of dissipative heating into hurricane energy budget, contains an additional physical error. It is correctly stated in the paper that “frictional dissipation of kinetic energy ultimately occurs at molecular scales”, with molecular friction forces correctly described by formula (1) of BE1998

$$\frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right), \quad (\text{BE1})$$

where  $\nu$  is the molecular kinematic viscosity;  $\nu \sim u_m l_m \sim 10^{-5} \text{ m}^2 \text{ s}^{-1}$ , where  $u_m \sim 500 \text{ m s}^{-1}$  is velocity of molecules,  $l_m \sim 10^{-7}$  is the mean free path length of air molecules. It is well-known that molecular kinematic viscosity is  $10^7$  times smaller than the eddy viscosity  $\nu_e$ , which in hurricanes is of the order of  $\nu_e \sim u_z h \sim 10^2 \text{ m}^2 \text{ s}^{-1}$ , i.e.  $\nu/\nu_e \sim 10^{-7}$ . For this reason the molecular friction forces that correspond to energy dissipation into thermal energy, are by the same amount smaller than the turbulent friction forces, these unrelated to dissipation into the thermal energy of chaotic molecular motion. As far as the linear scale of hurricane velocity change is macroscopic and is of the order of atmospheric height scale  $h$ , molecular friction forces are of the order of  $\nu u/h^2$ . Formula (5) used in BE1998 is

$$\nu \frac{\partial u_i}{\partial x_3} \Big|_0 = C_D u_i \sqrt{u_1^2 + u_2^2}. \quad (\text{BE5})$$

Hurricane velocities are of approximately one and the same order of magnitude,  $u_i \sim u_1 \sim u_2 \sim u$ , the scale of  $u_i$  change is of the order of  $h$ , consequently, from (BE5) we have  $\nu \frac{\partial u_i}{\partial x_3} \Big|_0 \sim \nu u / h \sim C_D u^2$ . Since  $C_D \sim u_z / u$ ,

we have  $\nu \sim C_D u_z h \sim 10^2 \text{ m}^2 \text{ s}^{-1}$ , which means that instead of molecular kinematic viscosity in all subsequent formulae of BE1998 it is eddy viscosity that is used.

As already noted, eddy viscosity and turbulent friction forces characterize transformation of the kinetic energy of large macroscopic eddies into kinetic energy of smaller, yet also macroscopic, eddies. Eddy viscosity does not describe conversion of kinetic energy to heat; for this reason it cannot be used in the estimates of dissipative heating (e.g., Businger, Businger, 2001). In the result of the replacement of molecular kinematic viscosity by eddy viscosity in BE1998 and subsequent papers the contribution of dissipative heating into hurricane energy budget was overestimated by about  $10^7$  times.

We will now list the main physical inconsistencies in the theoretical considerations of hurricanes presented in E1986, E1991, E1995, E2003, E2005, E2006 and BE1998.

1) A misconception inherent to modern meteorology as a whole is the assumption of the hydrostatic equilibrium of moist air. In the result, the pressure drop of moist air that occurs due to water vapor condensation, which, as we have shown, is the driver of hurricane motions, is neglected.

2) No quantitative account is made of the heat flux emitted to space from the area occupied by the hurricane. In the meantime, this flux is thousands of times smaller than the one necessary for the explanation of hurricane's power on the basis of Carnot's heat engine (E1986, E1991, E1995, E20003, E2005, E2006, BE1998).

3) In the consideration of Carnot's cycle, energy arising during dissipation of work  $A$  is added to heat  $\Delta Q_s$ , allegedly consumed by the hurricane from the oceanic heat source, see (A25). This leads to the violation of both first and second laws of thermodynamics, see (A22) and (A23), (A24). The resulting expression for the mechanical work  $A$  is physically implausible, as it tends to infinity with temperature  $T_0$  of the heat sink approaching absolute zero, see formulae (22), (23) in BE1998, formulae (6)-(8) in E2003, and E2005, E2006.

4) No account is made for the difference in the volumes of the microscopic transition layer above the air-sea interface and the macroscopic atmospheric layer where the hurricane develops. The ratio of these volumes



is of the order of  $10^8$ . Instead, one uses the physically meaningless difference in mass-specific enthalpies between the two atmospheric layers (E1995).

5) While estimating the contribution of dissipative heating into the energy budget of the hurricane, eddy viscosity is used instead of molecular kinematic viscosity, the former exceeding the latter by  $10^7$  times, formulae (1), (5), (6) in BE1998.

Despite these errors, in all papers discussed a satisfactory agreement between the theoretical results and empirical data was demonstrated, although the former violated the fundamental physical laws. This was possible due to the fact that the agreement was implicitly sought not between the proposed incorrect theory and the data, but between the data and the same data combined with use of the proposed mathematical formulae. For example, the difference of mass-specific enthalpies between the atmosphere and the microscopic transition layer is, as discussed above, physically irrelevant to hurricane energy budget, yet it numerically coincides with the observed pressure gradient and, hence, can be used to calculate a plausible value of hurricane velocity (E1995). Similarly, neither of the terms of Eq. (A21) describes transformation of thermal energy into kinetic one or vice versa, but they are all of the order of the major hurricane energy budget terms. Then if the dissipative heating is mistakenly calculated as turbulent friction, it can be concluded that dissipative influence hurricane energetics (BE1995). Such an approach is common to many modeling studies in modern meteorology. Such models do not contain any additional information as compared to the empirical data used and are deprived of predictive power.

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