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**Theoretically predicted and empirically observed  
dependences of the terrestrial greenhouse effect  
on temperature: Implications for climate stability**

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**Теоретически предсказываемые и эмпирически наблюдаемые зависимости  
глобального парникового эффекта от температуры  
и их влияние на устойчивость климата**

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**Аннотация**

Получено приближенное уравнение распространения теплового излучения в атмосфере, решение которого совпадает с точным решением известного уравнения радиационного переноса в условиях радиационного равновесия с относительной точностью порядка 20% при любых значениях оптической глубины атмосферы. Найденное уравнение позволяет просто учесть нерадиационные потоки тепла (скрытой теплоты и конвекции). Решение полученного уравнения дает аналитическую зависимость парникового эффекта от концентраций произвольного набора парниковых веществ. С помощью найденной зависимости парникового эффекта от концентрации парниковых веществ и температуры поверхности проанализирована устойчивость существующего климата Земли. Показано, что существующий климат Земли с жидкой гидросферой физически неустойчив. Количественно оценены наблюдаемые отклонения в поведении парникового эффекта от теоретических физических закономерностей. Высказаны количественные аргументы в пользу утверждения о том, что устойчивость существующего климата может поддерживаться управляющим действием естественной биоты Земли.

**Theoretically predicted and empirically observed dependences  
of the terrestrial greenhouse effect on temperature:  
Implications for climate stability**

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**Abstract**

We derive an approximate equation for radiative transfer in an atmosphere containing greenhouse substances. Its solution coincides with the exact solution of the conventional radiative transfer equation to the accuracy of 20% for all values of atmospheric optical depth. The derived equation makes it possible to easily take into account the non-radiative thermal fluxes (convection and latent heat) and obtain an analytical dependence of the greenhouse effect on atmospheric concentrations of greenhouse substances, to be used in the analysis of modern climate stability. It is shown that the modern value of global mean surface temperature, which corresponds to the liquid state of the terrestrial hydrosphere, is physically unstable. The observed stability of modern climate over geological timescales is due to dynamic singularities in the temperature-dependent behaviour of the greenhouse effect, which we here quantify. We hypothesise that such singularities appear due to controlling functioning of the natural global biota and discuss major quantitative arguments in support of this conclusion.

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## Introduction

Stability of a planetary climate is determined by the values and temperature-dependent behaviour of the planetary albedo and atmospheric greenhouse effect. At zero albedo and in the absence of the greenhouse effect, the temperature of the planet's surface is dictated by the incoming flux of solar energy, i.e. by the orbital position the planet occupies in the solar system. Below this temperature is called orbital. Values of albedo common to planets of the solar system correspond to decrease of the surface temperature by no more than several tens of degrees Kelvin. By contrast, the greenhouse effect may increase surface temperature by several hundred degrees Kelvin, as it takes place on Venus (Mitchell, 1989).

The Earth's orbital temperature is equal to  $+5^{\circ}\text{C}$ . The Earth's albedo, which is likely to approach its minimum possible value for ordinary planetary surfaces, would, in the absence of a greenhouse effect, lower the surface temperature down to  $-18^{\circ}\text{C}$ . The modern greenhouse effect increases the global mean surface temperature of the planet by  $33^{\circ}\text{C}$ , up to  $+15^{\circ}\text{C}$ .

The overwhelming part of the greenhouse effect on Earth is determined by atmospheric water vapour, cloudiness and atmospheric  $\text{CO}_2$ . Due to the presence of large amounts of liquid water on the planet's surface, the atmospheric water content grows exponentially with increasing surface temperature. According to the Clausius-Clapeyron equation, this growth results in doubling of atmospheric water content for each ten degrees of temperature increase. It cannot be excluded *a priori* that this positive feedback will lead to an unlimited increase of the greenhouse effect and surface temperature, until the oceans evaporate completely. The possibility of such a "runaway" greenhouse effect has been repeatedly discussed in the literature (Ingersoll, 1976; Rasool, de Berg, 1979; Nakajima et al., 1992; Weaver, Ramanathan, 1995).

Choosing values of albedo and greenhouse effect, it is possible to equate the incoming flux of short-wave solar radiation absorbed by the planet and the outgoing flux of long-wave radiation emitted by the planet into space. This equality will determine a stationary equilibrium temperature of the Earth's surface. However, due to the positive feedback outlined above, such an equilibrium may appear to be unstable. Any small fluctuations will be then able to drive the surface temperature either in the direction of cooling, towards the planet's glaciation, or in the direction of warming, towards complete evaporation of the hydrosphere.

In the studies of the runaway greenhouse effect (Ingersoll, 1976; Rasool, de Berg, 1979; Nakajima et al., 1992; Weaver, Ramanathan, 1995) one addresses the problem of whether the existence of an equilibrium surface temperature at different values of solar constant is possible or not. Stability of the equilibrium temperature, should such an equilibrium exist, is rarely dis-

cussed. The possibility of an equilibrium global mean surface temperature being unstable, was mentioned by Ingersoll (1967).

Modern climate of Earth is stable, as testified for by the observations that oscillations of the global mean surface temperature did not exceed  $10^\circ\text{C}$  during the last several hundred million years (Berggren, Van Couvering, 1986), several degrees Celsius — during the last ten thousand years, and several fractions of degree Celsius — during the last century (Savin, 1977; Watts, 1982). It follows that in the vicinity of the modern value of the global mean surface temperature there are certain negative feedbacks in action, that overcome the positive greenhouse feedback discussed above (Gorshkov, 2001).

These negative feedbacks are manifested as empirically established regularities in the temperature-dependent behaviour of various climate-forming factors. Climate models that incorporate these regularities yield, as expected, a stable equilibrium surface temperature (Manabe, Wetherald, 1967; North, Coakley, 1979; North et al., 1981; Dickinson, 1985). In the meantime, there are no theoretical studies that would predict or explain the observed stability of the modern Earth's climate *a priori*, on the basis of the known physical laws.

This paper aims at establishing a theoretical dependence of the greenhouse effect on atmospheric concentrations of greenhouse substances, which is then used in the analysis of the nature of stability of the modern Earth's climate. In Section 1 we show that the equation of transfer of long-wave radiation in a given spectral interval can be reasonably approximated by an equation of the diffusion (heat conductivity) type at all values of atmospheric optical depth  $\tau$ . In the case of radiative equilibrium, solution of the corresponding diffusion equation represents a linear dependence of the upward flux of long-wave radiation at the surface on the atmospheric optical thickness  $\tau_s$ , which differs from the exact solution of the radiative transfer equation by no more than 20% for all values of  $\tau_s$ .

In Section 2 we employ standard methods for accounting in the diffusion equation for non-radiative thermal fluxes of convection and latent heat. It is shown that such an account retains the linear dependence of the upward flux of thermal radiation at the surface on the atmospheric optical thickness  $\tau_s$  in a definite spectral interval. It is only the slope of the corresponding line that is changed. In Section 3 we derive an analytical formula for the dependence of the greenhouse effect (defined here as the difference

between thermal fluxes at the surface and outside the atmosphere) on atmospheric concentrations of greenhouse substances and use it in the theoretical analysis of stability of possible Earth's climates. It is shown that a climate with liquid hydrosphere is physically unstable. In Section 4 the available empirical data are employed to quantify deviations of the real climate-governing factors from the theoretical physical behaviour, that are necessary to explain the observed stability of modern climate of Earth. In Section 5 (Conclusions) we analyse possible reasons for the observed climate stability and hypothesise that it is due to the controlling functioning of the global natural biota (Gorshkov et al., 2000). We finally give quantitative arguments in support of this hypothesis.

### 1. Analysis of the radiative transfer equation.

In the planar three-dimensional case, which corresponds to averaging over longitude and latitude, equation for transfer of radiation of a given wave length has the form (Michalas and Michalas, 1984):

$$\mu \frac{\partial I(\mu, z)}{\partial z} = -\frac{1}{l(z)} I(\mu, z) + \frac{S(z)}{l(z)}, \quad (1.1)$$

where  $I(\mu, z)$  is the intensity of radiation, i.e. the energy transported per unit time in a given direction  $\mathbf{n}$  through unit perpendicular area by a bundle of rays propagating within an infinitely small solid angle (Milne, 1930, p. 72);  $\mu$  is the cosine between the given direction  $\mathbf{n}$  and the vertical axis  $z$ ,  $S(z)$ , the source function, is the radiative energy emitted per unit time in a cylinder of unit cross-section and length equal to the mean free path length  $l(z)$ . Function  $S(z)$  is assumed to be isotropic, i.e.  $\mu$ -independent. The first term in the right-hand part of (1.1) describes absorption of radiation in the corresponding volume, the second one describes emission of radiation within the same volume. Thus, eq. (1.1) represents the law of energy conservation: change in the energy of the ray,  $\partial I(\mu, z)$ , as the ray travels path  $\partial s = \partial z/\mu$ , is equal to the difference between the amounts of radiation absorbed and emitted by the matter along that path.

According to (1.1), in free space, where both absorption ( $l(z) = \infty$ ) and emission ( $S(z) = 0$ ) are absent, the intensity should remain constant along the path of any ray. This is indeed the case if the intensity  $I(\mu, z)$  is defined, as above, as the mathematical limit for an infinitely small solid angle (Milne, 1930, p. 72). If the intensity is defined for any finite unit of solid angle, however small (see, e.g., Goody and Young, 1989, p. 16), eq. (1.1) does not hold for free space. (This can be easily shown on the example of a point source of radiation, for which the flux of energy through a finite solid angle decreases as the bundle of emitted rays diverges in space.) As far as all physical measurements are characterised by finite sensitivity, the intensity (1.1), defined for an

infinitely small solid angle, appears to be a variable that cannot be measured directly. Measurable values are those obtained by integrating (1.1) over solid angle, which, in the planar case considered, corresponds to integrating over  $\mu$ . For our purposes it is enough to consider the radiation flux  $F$  and the energy density  $E$ . In the case of radiative equilibrium (absence of energy exchange between the matter and radiation) these are related to intensity (1.1) as follows:

$$\frac{Ec}{4\pi} \equiv J, \quad \frac{F}{4\pi} \equiv H, \quad S(z) = J(z) \equiv \frac{1}{2} \int_{-1}^1 I(z, \mu) d\mu, \quad H(z) \equiv \frac{1}{2} \int_{-1}^1 \mu I(z, \mu) d\mu \quad (1.2)$$

In this section we will refer to the mathematically convenient variables  $J$  and  $H$  (so-called Eddington variables) as to the normalised energy density and normalised flux, respectively.

In order to obtain equations describing radiative transfer in terms of directly measurable variables, it is therefore necessary to integrate (1.1) over  $\mu$ . However, this will not yield an equation constraining  $H$  or  $J$  separately, but only an equation relating the two variables.

The problem is solved by representing eq. (1.1) in an integral form (Chandrasekhar, 1950; Michalas and Michalas, 1984; Goody and Yung, 1989), first replacing in (1.1) height  $z$  by optical depth  $\tau$ ,  $dz/l(z) \equiv -d\tau$ ,  $\tau \equiv \int_z^\infty \frac{dz}{l(z)}$ . Multiplying then both parts of (1.1) by  $e^{-\tau\mu}$  (this factor represents the

solution of the homogenous eq. (1.1) at  $S(z) = 0$ ) and grouping together the two intensity-dependent terms of (1.1) at the left-hand part of the equation, one obtains that the  $\tau$ -derivative of  $I(\mu, \tau)e^{-\tau\mu}$  is equal to  $S(\tau)e^{-\tau\mu}$ . Then, one integrates both parts of the new equation over  $\tau$  within the limits  $\tau \leq \tau' < \infty$  for  $\mu > 0$  and within the limits  $0 \leq \tau' < \tau$  for  $\mu < 0$  and takes into consideration the boundary conditions — absence of exponential growth of  $H$  and  $J$  at  $\tau \rightarrow \infty$  for  $\mu > 0$  and zero value of the intensity at  $\tau = 0$  for  $\mu < 0$  (Michalas and Michalas, 1984). In the two relations for  $\mu I(\tau, \mu)$  thus obtained, for  $\mu > 0$  and for  $\mu < 0$ , one may further introduce new integration variables,  $\tau' - \tau = \mu x$  for  $\mu > 0$  and  $\mu' = -\mu$  and  $\tau' - \tau = \mu' x$  for  $\mu = -\mu' < 0$ , and integrate them taking into account (1.2). Then, dropping the prime of the new integration variable  $\mu'$ , the following relations for  $J(\tau)$  and  $H(\tau)$  are obtained:

$$J(\tau) = J^+(\tau) + J^-(\tau), \quad H(\tau) = H^+(\tau) - H^-(\tau), \quad (1.3a)$$

$$J^+(\tau) \equiv \frac{1}{2} \int_0^1 d\mu \int_0^\infty e^{-x} J(\tau + \mu x) dx, \quad J^-(\tau) \equiv \frac{1}{2} \int_0^1 d\mu \int_0^{\tau/\mu} e^{-x} J(\tau - \mu x) dx, \quad (1.3b)$$

$$H^+(\tau) \equiv \frac{1}{2} \int_0^1 \mu d\mu \int_0^\infty e^{-x} J(\tau + \mu x) dx, \quad H^-(\tau) \equiv \frac{1}{2} \int_0^1 \mu d\mu \int_0^{\tau/\mu} e^{-x} J(\tau - \mu x) dx, \quad (1.3c)$$

$$\frac{dH^+(\tau)}{d\tau} = \frac{dH^-(\tau)}{d\tau} = \frac{1}{2}[J^+(\tau) - J^-(\tau)], \quad (1.3d)$$

$$J(\tau=0) = H(\tau=0) = 0, \quad (1.3e)$$

where + and – refer to the normalised fluxes and energy density of upwelling (+) and downwelling (–) radiation, respectively. Eq. (1.3d) can be obtained differentiating the integral part of (1.3c) over  $\tau$ , using integration by parts and expressions (1.3a), (1.3b). Eqs. (1.3) are exact, with no approximations made. Integral eqs. (1.3b), (1.3c) differ from the Schwarzschild-Milne equations (Michalas and Michalas, 1984) by replacement of variables inside the integrals.

In the one-dimensional case, when the radiation propagates in the form of parallel rays exactly up or down, one has to put  $\mu = 1$  and omit integration over  $\mu$  in (1.3):

$$J^+(\tau) = H^+(\tau) = \frac{1}{2} \int_0^\infty e^{-x} J(\tau+x) dx, \quad J^-(\tau) = H^-(\tau) = \frac{1}{2} \int_0^\tau e^{-x} J(\tau-x) dx. \quad (1.4)$$

The greenhouse effect is fully determined by the changes that the upwelling radiation flux  $H^+(\tau)$  undergoes propagating from the Earth's surface ( $\tau = \tau_s$ ) to the top of the atmosphere ( $\tau = 0$ ). Thus, we are primarily interested in finding the dependence of  $H^+(\tau)$  on  $\tau$  from (1.3c).

It follows from (1.3d) that  $dH(\tau)/d\tau = 0$  and, consequently,  $H(\tau) = H = \text{const}$ . Using (1.3a,b,c) it can be shown that this condition can be only fulfilled, if at large values of  $\tau \gg 1$  the normalised energy density  $J(\tau)$  grows proportionally to the first power of  $\tau$ , i.e. if at  $\tau \gg 1$  we have  $J(\tau) = c_J H \tau + o(\tau^{n-1})$ , where  $c_J = \text{const}$ . Indeed, let us express  $J(\tau)$  as  $J(\tau) = c_J H \tau^n$ , where  $n$  is an arbitrary power, and put this expression into the integrals (1.3c). The sum of the two integrals (1.3c) makes  $H(\tau)$ . Expanding the integral terms  $(\tau \pm \mu x)^n$  into the series of powers of  $\tau$ , summing the terms with equal power and performing the integration, one obtains the following leading term of the asymptotic expression for  $H(\tau)$  at large  $\tau$ :  $H(\tau) = (c_J / 3) H \tau^{n-1} + o(\tau^{n-3})$ . From this we both derive the value of  $c_J = 3$  and conclude that a constant non-zero value of  $H(\tau)$  is only possible when  $n = 1$ .

Let us now evaluate  $H^+(\tau)$  at  $\tau \gg 1$  using the first eq. (1.3c). We note that the major contribution into the integral for  $H^+(\tau)$  (1.3c) comes from the interval  $\mu x \sim 1$ . We put the known asymptotic expression for  $J(\tau)$  at  $\tau \gg 1$  into the first eq. (1.3c),  $J(\tau) = 3H(\tau + 0.710)$  and perform the elementary integration over  $\mu$  and  $x$ . The constant asymptotic term 0.710 was obtained by Hopf, see (Michalas and Michalas, 1984). One thus obtains:

$$f(\tau) \equiv H^+(\tau)/H = 1.033 + \frac{3}{4} \tau, \quad q'(\tau) = \frac{3}{4},$$

$$H^\pm(\tau) = J^\pm(\tau)/2, \quad \tau \gg 1. \quad (1.5)$$

Let us now explore the behaviour of  $f(\tau)$  at small  $\tau$ ,  $\tau \ll 1$ . From (1.3a), (1.3d) and (1.3e) we have  $[H^+(0)]' = \frac{1}{2}J(0)$ . In the planar three-dimensional case, at  $\tau = 0$  photons propagate isotropically into the upper hemisphere only. The squared velocity of photons at  $\tau = 0$  is determined by the relation  $c^2 = c_x^2 + c_y^2 + c_z^2 = 3 c_z^2$ , as far as all directions of movement of isotropically propagating photons are equally probable. We have therefore  $c_z = c/\sqrt{3}$ , which means that the flux  $F$  is related to energy density  $E$  as  $F = Ec_z = Ec/\sqrt{3}$ . Finally, we obtain from (1.2) that  $J(0) = \sqrt{3}H$ . Naturally, the same result is obtained when eqs. (1.3) are solved exactly (Michalas and Michalas, 1984, p. 357). Thus,  $f'(0) = \sqrt{3}/2$  and in the region of small  $\tau$  we have:

$$f(\tau) = 1 + \frac{\sqrt{3}}{2}\tau, \quad f'(\tau) = \sqrt{3}/2, \quad \tau \ll 1. \quad (1.6)$$

In the intermediate region of  $\tau \sim 1$  function  $f(\tau) \equiv H^+(\tau)/H$  does not allow for a simple analytical representation. Nevertheless, it is clear that within this region  $f(\tau)$  remains of the order of unity. At all  $\tau$  the first derivative of  $f(\tau)$  remains positive and the second derivative of  $f(\tau)$  remains negative, see (1.3d), as far as  $J^+(\tau) > J(\tau)$  and  $[J^+(\tau)]' > [J(\tau)]'$  for all  $\tau$ . Consequently,  $f(\tau)$  increases, while  $f'(\tau)$  decreases monotonously to their asymptotic values given by (1.5).

In the one-dimensional case one can similarly obtain the exact solution of (1.4) valid for all  $\tau$

$$f(\tau) = 1 + \frac{1}{2}\tau, \quad 0 \leq \tau < \infty. \quad (1.7)$$

This solution is physically transparent. In the one-dimensional isotropic case the resonance scattering of a photon propagating in the upward direction may with an equal probability result in either upward or downward emission. The upward scattering of such a photon is equivalent to the absence of interaction with the matter. Downward scattering (i.e. scattering resulting in a change of the initial direction of photon movement) occurs on average every time the photon covers two mean free path lengths. In effect, this is equivalent to the twofold decrease of the optical path  $\tau$ .

Eqs. (1.5), (1.6) for the three-dimensional planar case can be written as

$$f(\tau) = 1 + \tilde{\tau}, \quad \tilde{\tau} \equiv k\tau, \quad (1.8)$$

where  $\frac{\sqrt{3}}{2} \leq k \leq \frac{3}{4}$  for any  $\tau$ ,  $0 \leq \tau < \infty$ . The 13% change that coefficient  $k$

undergoes in the region  $\tau \sim \tau_c = 0.28$ , from  $k \approx 3/4$  at  $\tau \gg 1$  to  $k \approx \sqrt{3}/4$  at  $\tau \ll 1$ , is dictated by the change in the character of radiation propagation. At large values of  $\tau$  the radiation propagates approximately equally into the upper and lower hemispheres ( $H^+ \approx H^-$ ,  $J^+ \approx J^- \approx 2H^{\pm}$ ), while at small  $\tau$  the predominant direction is into the upper hemisphere only ( $H^+ \gg H^-$ ,  $J^+ \gg J^-$ ,  $J^+ \approx \sqrt{3} H^+ \approx \sqrt{3} H$ ). In other words, the change in  $k$  monitors the corresponding change in the relation between the upwelling flux  $H^+$  and the energy density  $J$ . Consequently, we can consider  $k$  as a constant renormalisation of  $\tau$  (or mean free path  $l$ ) and omit the wave above  $\tilde{\tau}$  in the relation for  $f(\tau)$  (1.8).

We can now write the following diffusion-type equations for  $f(\tau)$  (Makarieva, Gorshkov, 2001):

$$\frac{df(\tau)}{d\tau} = 1, \quad -\frac{d^2 f(\tau)}{d\tau^2} = \delta(\tau - \tau_s), \quad \tau_s = \int_0^{\infty} \frac{dz}{l(z)}, \quad (1.9)$$

where the second equation (1.9) takes into account zero thermal radiation flux at  $z < 0$  (under ground).

## 2. An account of non-radiative heat fluxes

Now we extend our consideration to the case when there is a non-radiative dynamic flux of heat in the atmosphere, which forms due to convective transport of latent heat of water vapor and internal energy of air molecules. The only way for this heat flux to be released into space is by means of excitation at its expense of an additional number of energy levels of molecules of greenhouse substances and subsequent emission of thermal photons by the excited molecules. Additional energy levels are excited when molecules of greenhouse substances collide with molecules of other air constituents.

To take the non-radiative fluxes into account, we add to the right-hand part of the second equation (1.9) a positive term describing the density of the flux of non-radiative excitation of greenhouse molecules per unit optical depth, after which eqs. (1.9) and their solution can be re-written as follows:

$$-\frac{d^2}{d\tau^2} f(\tau) = \gamma_s \delta(\tau - \tau_s) + \alpha(\tau), \quad j(\tau) \equiv \frac{dq(\tau)}{d\tau} = \gamma_s + \int_{\tau}^{\tau_s} \alpha(\tau') d\tau', \quad (2.1a)$$

where

$$\tau \equiv \int_z^{\infty} \frac{dz'}{l(z')}, \quad d\tau = -\frac{dz}{l(z)}, \quad \tau_s \equiv \int_0^{\infty} \frac{dz}{l(z)} \equiv \frac{h}{l(0)}, \quad j(\tau=0) = f(\tau=0) = 1; \quad (2.1b)$$

$$\gamma_s + \int_0^{\tau_s} \alpha(\tau) d\tau = 1; \quad (2.1c)$$

$$f(\tau) = 1 + \int_0^\tau j(\tau') d\tau', \quad f(\tau_s) = 1 + \gamma_s \tau_s + \int_0^{\tau_s} d\tau \int_\tau^{\tau_s} d\tau' \alpha(\tau'). \quad (2.1d)$$

At a given layer  $\tau$ , variable  $j(\tau)$  represents the rate of change with  $\tau$  of the flux of thermal radiation scaled by  $F_e$ . It includes the non-radiative thermal fluxes that have converted into it at the layers located below  $\tau$ . Function  $\alpha(\tau) > 0$  is the flux of the non-radiative excitation of molecules of greenhouse substances per one mean free path length of thermal photons. Eqs. (2.1a)-(2.1d) are written for the case of one greenhouse substance and may be extended to the general case of several greenhouse substances (Makarieva, Gorshkov, 2001).

The important condition  $\alpha_k > 0$  means that all the dynamic energy fluxes must be converted into the energy of thermal photons and not vice versa, which is a manifestation of the second law of thermodynamics. If the collisional excitation of the absorption bands of greenhouse substances were absent or negligibly small, the air temperature would be the same throughout the entire atmospheric column, coinciding with that of the Earth's surface. The brightness temperature in each spectral interval containing absorption bands of greenhouse substances coincides with the surface temperature at the surface and then decreases with height. This means that the population density of the excited states of greenhouse molecules decreases with height. If one now switches on collisional interaction, this will lead to excitation of additional molecules at the expense of thermal energy of the hotter air. As a result, the air temperature will drop, while the brightness temperature in the corresponding absorption intervals will increase. According to the second law of thermodynamics, thermal energy is transported from the hotter medium to the cooler, i.e. from air molecules to the absorption bands of the greenhouse substances, and not vice versa. In a stationary state, loss of energy by air molecules due to collisional excitation of the absorption bands is compensated by the non-radiative energy fluxes of convection and latent heat.

It follows from (2.1c) and (2.1d) that the non-radiative heat fluxes work to diminish the value of  $f(0)$  and the greenhouse effect. However, even in the limiting case of  $\gamma_s = 0$  (i.e. when the thermal flux from the Earth's surface is completely non-radiative), the greenhouse effect retains a non-zero value. This quite expected result is a manifestation of the fact that the convection itself may only arise when there is a non-zero greenhouse effect.

Using sum rule (2.1c) we can write the dependence of  $f(0)$  on  $\tau_s$  in the following form for any  $\tau_s$ :

$$f(\tau_s) = 1 + \tilde{\tau}_s, \quad \tilde{\tau}_s = k \tau_s, \quad (2.2)$$

$$k \equiv \overline{j(\tau_s)} \equiv \frac{1}{\tau_s} \int_0^{\tau_s} j(\tau) d\tau = \gamma_s + \frac{1}{\tau_s} \int_0^{\tau_s} d\tau \int_\tau^{\tau_s} d\tau' \alpha(\tau'),$$

$$0 \leq \gamma_s \leq k \equiv \overline{j(\tau_s)} \leq 1 \quad (2.3)$$

It is easy to check that the sum rule (2.1c) gives  $\overline{j(\tau_s)} = 1 + o(\tau_s)$  for  $\tau_s \ll 1$ . At  $\tau_s \gg 1$ , the averaged rate of the change of radiative flux,  $\overline{j(\tau_s)}$ , remains to be bounded from below by a finite value, irrespectively of the value of  $\gamma_s$ . This means that eqs. (1.9) for the greenhouse effect are valid for all cases, if one replace  $\tau_s$  by  $\tilde{\tau}_s$ . Such a replacement is equivalent to multiplying the mean free path length  $l$  by a constant factor greater than unity. This factor can be retrieved from experimental data. Thus, the account of convection retains the linear dependence of  $f(\tau_s) \equiv F_s/F_e$  on the optical thickness  $\tau_s$ . It is only the slope of the corresponding line that is changed (diminished). This result is in full agreement with the results of previous studies (see, e.g. work of Stephens, Greenwald (1991a) and their Fig. 7).

We give here values of  $f(\tau_s)$  for several model functions  $\alpha(\tau)$  that conform to the sum rule (2.1c). For the case when the dissipation of non-radiative fluxes is governed by a power law,  $\alpha(\tau) = \beta(\tau_s - \tau)^{n-1}$ , we have  $\beta = (1 - \gamma_s)n/\tau_s$  and  $f(\tau_s) = 1 + \tau_s(1 + n\gamma_s)/(1+n)$ . As far as  $\alpha(\tau)$  cannot increase with height, it is reasonable to assume that  $n \leq 1$ . For the case of the non-radiative dissipation exponentially decreasing with height,  $\alpha(\tau) = \beta \exp[(\tau_s - \tau)/\tau_\alpha]$ , we have  $\beta = (1 - \gamma_s)/\tau_\alpha(1 - \exp[-\tau_s/\tau_\alpha])$  and  $f(\tau_s) = 1 + \tau_s(1 + \gamma_s)/2$  for  $\tau/\tau_\alpha \ll 1$  and  $f(\tau_s) = 1 + \tau_s$  for  $\tau/\tau_\alpha \gg 1$ . In the latter case the account of convection does not have any impact on the result obtained in the absence of convection. This is natural, because in the case of exponential decrease of convective fluxes with height, the major part of dissipation takes place in the lower atmospheric layers, so that the major contribution into the integral (2.3) comes from the interval  $(\tau_s - \tau)/\tau_\alpha \sim 1$ , i.e.  $\tau_s - \tau \ll \tau_s$ . In all other cases the coefficient  $k$  (2.2) at the first power of  $\tau_s$  changes by no more than twofold as compared to unity corresponding to the absence of convection.

### 3. Stability of possible Earth's climates

Averaged over a sufficiently large area, the energy balance at the Earth's surface (including the atmosphere) has the form (North et al., 1981; Dickinson, 1985):

$$C \frac{dT}{dt} = F_{\text{in}} - F_{\text{out}} \equiv -\frac{dU}{dT}, \quad (3.1)$$

$$F_{\text{in}} \equiv I_a a(T), \quad F_{\text{out}} \equiv \sigma_R T^4 b(T),$$

Here  $T$  is the absolute temperature of the Earth's surface;  $C$  is the average heat capacity per unit surface area;  $F_{\text{in}}$  is the flux of short-wave solar radiation absorbed by the Earth's surface,  $I_a$  is the total incoming flux of solar radiation outside the atmosphere, the global mean value of  $I_a$  is equal to  $\overline{I_a} = I/4$ ,

where  $I = 1367 \text{ W m}^{-2}$  is the solar constant (Wilson, 1984; Mitchell, 1989);  $a(T) \equiv 1 - A(T)$  is the share of the incoming flux of solar radiation absorbed by the Earth's surface (coalbedo);  $A(T)$  is the planetary albedo;  $F_{\text{out}}$  is the flux of long-wave thermal radiation leaving the planet into space;  $\sigma_R = 5,67 \cdot 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$  is the Stephen-Boltzmann constant;  $b(T) \equiv F_{\text{out}}/F_s$ , where  $F_s$  is the flux of thermal radiation from the Earth's surface, has the meaning of atmospheric transmissivity with respect to thermal radiation,  $b(\vartheta) = 1/f(\vartheta)$ , see (1.9);  $U(T)$  is the potential (Liapunov) function. The only independent variable in (3.1) is temperature  $T$ .

In a stationary state, when the energy content does not change,  $C \frac{dT}{dt} = 0$ , the derivative of the potential function  $U(T)$  turns to zero, and  $U(T)$  has an extreme—maximum or minimum. The central part of (3.1) also turns to zero, thus determining a stationary temperature  $T = T_s$ :

$$I_a a(T_s) - \sigma_R T_s^4 b(T_s) = - \left. \frac{dU}{dT} \right|_{T=T_s} = 0 \quad \text{or}$$

$$T_s = T_a \left( \frac{a(T_s)}{b(T_s)} \right)^{1/4}, \quad T_a \equiv \left( \frac{I_a}{\sigma_R} \right)^{1/4}, \quad \bar{T}_a \equiv T_o = \left( \frac{I}{4\sigma_R} \right)^{1/4} = 278 \quad (3.2)$$

Here  $T_o$  is the orbital temperature. The second derivative of the potential function  $U(T)$ ,  $W$ , determines the character of the extreme. It is a stable minimum when  $W > 0$ , and an unstable maximum when  $W < 0$ .

$$W \equiv \left( \frac{d^2U}{dT^2} \right)_{T=T_s} = \frac{I}{4} \left( \frac{a(T)}{T} (4 + \beta - \alpha) \right)_{T=T_s}, \quad \alpha \equiv \frac{da}{dT} \frac{T}{a}, \quad \beta \equiv \frac{db}{dT} \frac{T}{b}. \quad (3.3)$$

The stationary state is stable  $\alpha - \beta < 4$  and unstable at  $\alpha - \beta > 4$ . In particular, stationary states in the regions of slowly changing, practically constant  $a$  and  $b$ , for which  $\alpha \ll 4$  and  $\beta \ll 4$ , are stable.

On Earth the major greenhouse substances are the water vapour and carbon dioxide. The existing spectral windows remain transparent at clear sky and are “closed” with appearance of clouds.

Using (1.9), it is easy to show that for the terrestrial atmosphere the transmissivity  $b$  can be written as

$$b = \frac{\delta_{\text{H}_2\text{O}}}{\tau_{s0} + \tau_{s\text{H}_2\text{O}} + 1} + \frac{\delta_{\text{CO}_2}}{\tau_{s0} + \tau_{s\text{CO}_2} + 1} + \frac{\delta_0}{\tau_{s0} + 1}, \quad (3.4)$$

Here  $\tau_{si}$  represent the effective optical depth of the  $i$ -th greenhouse substance in the atmosphere, see (1.9);  $\delta_{\text{H}_2\text{O}}$  and  $\delta_{\text{CO}_2}$  are the relative spectral intervals that contain the major absorption bands of water vapour ( $\text{H}_2\text{O}$ ) and  $\text{CO}_2$ ;  $\delta_0$  is the relative spectral interval which corresponds to spectral windows where there are no absorbers except for the clouds. The relative spectral intervals  $\delta_i$

describe the share of thermal radiation of the Earth's surface at  $T = 288$  K that is absorbed within the corresponding absorption intervals. We estimate the relative spectral intervals  $\delta_{\text{CO}_2}$ ,  $\delta_0$  and  $\delta_{\text{H}_2\text{O}}$  as follows:

$$\delta_{\text{CO}_2} = 0.19, \quad \delta_0 = 0.25, \quad \delta_{\text{H}_2\text{O}} = 0.56, \quad \sum_i \delta_i = 1 \quad (3.5)$$

The value of  $\delta_{\text{CO}_2}$  is calculated for the major absorption band of  $\text{CO}_2$  centered at  $15 \mu\text{m}$  and extending from  $13 \mu\text{m}$  to  $17 \mu\text{m}$  (Rodgers, Walshaw, 1966). The atmospheric spectral window spreads from  $8 \mu\text{m}$  to  $12 \mu\text{m}$ . Its value (3.5) approximately coincides with the estimate given by Weaver and Ramanathan (1995). The absorption area of  $\text{H}_2\text{O}$ , located at different parts of the thermal spectrum (Rodgers, Walshaw, 1966; Goody, Yung, 1989), is calculated from the condition  $\sum_i \delta_i = 1$ .

The effective optical depth  $\tau_{si}$  in (3.4) is estimated as the relative difference between the upward fluxes of thermal radiation in the corresponding spectral interval at the Earth's surface,  $F_{si}^+$ , and outside the atmosphere,  $F_{ei}^+$ :

$$\tau_{si} \equiv (F_{si}^+ - F_{ei}^+) / F_{ei}^+ = k h_i n_i \sigma_i, \quad (3.6)$$

$$F_{si}^+ = \int_{\lambda_1}^{\lambda_2} I_P(\lambda, T_s) d\lambda, \quad F_{ei}^+ = \int_{\lambda_1}^{\lambda_2} I_P(\lambda, T_{ei}) d\lambda,$$

where  $\lambda_2 - \lambda_1 = \Delta\lambda_i$  is the spectral interval of wavelengths, which defines the value of  $\delta_i$ ;  $I_P(\lambda, T)$  is the Planck function,  $T_{ei}$  is the brightness temperature in the spectral interval  $\Delta\lambda_i$  outside the atmosphere;  $h_i$ ,  $\sigma_i$  and  $n_i$  are the height of the upper radiating layer of a homogeneous atmosphere, absorption cross-section and concentration of  $i$ -th greenhouse substances, respectively;  $F_{ei}$  is the radiative flux in the  $i$ -th spectral interval outside the atmosphere. In calculation of  $F_{ei}^+$  the use of Planck function is not indispensable. It is employed here for determination of the brightness temperature only. One could obtain  $F_{ei}^+$  by direct integration of the spectral distribution of the outgoing thermal flux outside the atmosphere.

The relative absorption intervals  $\delta_{\text{H}_2\text{O}}$ ,  $\delta_{\text{CO}_2}$  and  $\delta_0$  include contributions from other greenhouse gases. For instance, the relative absorption interval  $\delta_{\text{CO}_2}$  contains weak absorption bands of  $\text{H}_2\text{O}$ , while  $\delta_0$  contains absorption band of  $\text{O}_3$  and the so-called continuum absorption spectrum of  $\text{H}_2\text{O}$  (Rodgers, Walshaw, 1966; Goody, Yung, 1989). However, the optical depth  $\tau_{sk}$  corresponding to these weak absorption bands is small compared to the corresponding value  $\tau_{si}$  of the greenhouse substance making the major contribution into

the interval considered,  $\tau_{s k} \ll \tau_{s i}$ . Thus, such  $\tau_{s k}$  were neglected in (3.4).

As shown in Sections 1-2, the effective optical thickness  $\tau_{s i}$  (3.6) differs from the real optical thickness by a multiplier  $k \sim 1$  ( $0.5 < k < 1$ ). This multiplier accounts for the non-radiative heat fluxes and divergence of rays in the three-dimensional atmosphere, which causes distortion from the diffusion equation. The most important feature of eq. (3.6) is the direct proportionality between  $\tau_{s i}$  and the total mass  $h_i n_i$  of  $i$ -th greenhouse substance in the atmospheric column.

In accordance with Boltzmann distribution, height of the upper radiating layer of the homogeneous atmosphere,  $h_i$ , grows proportionally to the temperature of the Earth's surface. The atmospheric concentration of water vapour near the surface is determined by the equilibrium between the water vapour and liquid water of the global oceans and terrestrial vegetation. The total amount of water vapour in the atmospheric column,  $h_{\text{H}_2\text{O}} n_{\text{H}_2\text{O}}$ , and its optical thickness increase due to temperature dependences of both  $h_{\text{H}_2\text{O}}$  and  $n_{\text{H}_2\text{O}}$ . Surface temperature is related to the upward flux of thermal radiation of the surface by the Stephen-Boltzmann formula,  $F_s^+ = \sigma_R T^4$ . Thus, at large  $\tau_{s i} \gg 1$  we have from (3.6) that  $h_i \propto (F_{s i}^+)^{1/4} \propto (h_i n_i \sigma_i)^{1/4}$ , so that  $h_i \propto (n_i \sigma_i)^{1/3}$  and  $\tau_{s i} \propto (n_i \sigma_i)^{4/3}$ .

To construct the dependence of the global transmissivity function  $b$  on surface temperature  $T$  we will assume that, in accordance with observations, the modern global mean surface temperature  $T_s = 288$  K ( $15^\circ\text{C}$ ) is stationary, thus conforming to (3.2). Using (3.6), we determine the modern value of  $\tau_{s \text{CO}_2}$  from the directly measured values of  $F_{s \text{CO}_2}^+$  and  $F_{e \text{CO}_2}^+$  (Conrath et al., 1970; Goody and Yung, 1989, p. 219). It gives (the brightness temperature of the  $15 \mu\text{m}$   $\text{CO}_2$  band outside the atmosphere is about 220 K)

$$\tau_{s \text{CO}_2} = 1.9 \quad (3.7)$$

The value of  $b(T_s)$  in the stationary point  $T_s = 288$  K is determined from (3.2) using the observed value of coalbedo  $a(T_s) = 0.70$  (Mitchell, 1989) and the observed value of orbital Earth's temperature  $T_o$  (3.2). These values give  $b(T_s) = 0.61$ .

The relative contribution of clouds,  $d$ , into the global absolute greenhouse effect  $f \equiv F_s - F_e$  is about 18%,  $d = 0.18$  (Raval and Ramanathan, 1989; Stephens, Greenwald, 1991b). Accordingly, the global value of  $b$  for the clear sky is equal to  $b_{\text{cs}} = F_{\text{ecs}} / F_s = (F_e + (F_s - F_e)d) / F_s = 0.68$ , where  $F_{\text{ecs}} > F_e$  is the outgoing flux of thermal radiation outside the atmosphere in the absence of clouds. Absence of clouds corresponds to  $\tau_{s0} = 0$  in (3.4). Setting in (3.4)  $\tau_{s0} = 0$ ,  $b = b_{\text{cs}} = 0.68$  and  $\tau_{s \text{CO}_2} = 1.9$  (3.7) and taking into account (3.5), we obtain

$$\tau_{s\text{H}_2\text{O}}(T_s) = 0.53 \quad (3.8)$$

Finally, using estimates (3.7) and (3.8) in (3.4) for the global value of  $b(T_s) = 0.61$ , we obtain from (3.4) the following value of  $\tau_{s0}$ :

$$\tau_{s0}(T_s) = 0.15, \quad r \equiv \tau_{s0} / \tau_{s\text{H}_2\text{O}} = 0.29 \quad (3.9)$$

The values of  $\tau_{si}$  (3.7)-(3.9) are derived from observations and take therefore into account the contributions into heat transfer of the non-radiative thermal

fluxes, see Section 2.

Near-surface concentration of the water vapour changes proportionally to the saturated concentration, which latter grows exponentially with increasing temperature in accordance with the Clausius-Clapeyron formula (see, e.g., Raval and Ramanathan, 1989; Nakajima et al., 1992):

$$\tau_{s\text{H}_2\text{O}}(T) = \exp\left(\varepsilon - \frac{T_{\text{H}_2\text{O}}}{T}\right), \quad T_{\text{H}_2\text{O}} \equiv \frac{Q_{\text{H}_2\text{O}}}{R} = 5.3 \cdot 10^3 \text{ K}, \quad \varepsilon = 17.76, \quad (3.10)$$

where  $\varepsilon$  is determined from (3.8). We neglect here the additional acceleration in growth of  $\tau_{s\text{H}_2\text{O}}$  due to the above discussed dependence of  $h_{\text{H}_2\text{O}}$  on  $n_{\text{H}_2\text{O}}$ . This dependence will only increase the rate of  $b(T)$  change with changing temperature, thus enhancing our conclusion about physical instability of the modern climate, which we arrive at below. We assume also that the ratio between atmospheric concentrations of liquid water and water vapour remains constant over a sufficiently broad temperature interval, so that  $r$  in (3.9) can be held constant.

Due to the finite mass of the Earth's hydrosphere, the greenhouse effect on Earth cannot grow—while  $b$  cannot diminish—indefinitely with growing surface temperature. This can be taken into account by stopping the growth of  $\tau_{s\text{H}_2\text{O}}(T)$ , when  $b(T)$  reaches a certain minimum value  $b_{\min} = 0.01$ . The value of  $b_{\min}$  is chosen equal to the corresponding value of  $b$  on Venus, where the atmospheric pressure is of the same order of magnitude as it would be on Earth were its hydrosphere evaporate (Pollack et al, 1980; Mitchell, 1989). Thus, we arrive at the following expression for  $b(T)$ , Fig. 1:

$$b(T) = \frac{0.56}{1.29\varphi(T)+1} + \frac{0.19}{0.29\varphi(T)+2.9} + \frac{0.25}{0.29\varphi(T)+1},$$

$$\varphi(T) \equiv \exp\left(17.76 - \frac{5.3 \times 10^3}{T}\right), \quad T \leq 422 \text{ K}$$

$$b(T) = 0.01, \quad T \geq 422 \text{ K} \quad (3.11)$$

At the modern value of the global mean surface temperature  $T_s = 15 \text{ }^\circ\text{C}$  the coalbedo function describing absorption of solar radiation by the Earth, approaches its maximum value (North et al., 1981). At colder temperatures, with increasing degree of the planet's glaciation, the coalbedo diminishes, while the albedo  $A(T)$  starts to grow. This growth is limited from above by the value  $A_{\max} \sim 0.7$  ( $a_{\min} \sim 0.3$ ), which characterises the reflectivity of snow cover (Hibler, 1985). With rising surface temperature,  $T > 15 \text{ }^\circ\text{C}$ , accompanied by evaporation of water, the albedo should increase as well due to the growing cloudiness and increasing atmospheric density. This is in agreement with the known high value of albedo on Venus, where  $A \sim 0.75$  (Mitchell, 1989), which approximately coincides with  $A_{\max} \sim 0.7$  for an ice-covered Earth. Modern

climate sensitivity with respect to temperature-dependent changes in albedo,  $\lambda_A \equiv -F_s a'(288 \text{ K})$ , is of the order of  $-(0.3 \div 0.8) \text{ W m}^{-2} \text{ K}^{-1}$  (Dickinson, 1985). This allows to conclude that the modern value of coalbedo is located to the left — along the  $T$  axis — from its maximum possible value (were the modern coalbedo coincide with the maximum, the climate sensitivity  $\lambda_A$ , which is proportional to the temperature derivative of coalbedo, would be equal to zero).

To take into account these physically transparent properties of coalbedo, we choose function  $a(T)$  in the form of a Gaussian curve. The location of the maximum is specified by the assumption that the global ice shield completely disappears when the global mean surface temperature rises up to 295 K (22°C). The modern albedo of Earth is equal to 0.30, where 0.25 falls on reflectivity of short-wave radiation by the atmosphere and 0.05 (one sixth part) is attributed to reflectivity by the Earth's surface, including the oceans (Schneider, 1989; Mitchell, 1989). Assuming that the global snow cover occupies about 5% of the total Earth's surface and taking the global mean albedo of the Earth's surface in the absence of clouds equal to  $\sim 0.1$  (Ramanathan, Coakley, 1978) and the albedo of snow about  $\sim 0.7$  (Hibler, 1985), we find that melting of the global snow cover will decrease the planetary albedo by  $(0.7 - 0.1) \cdot 0.05 \cdot (1/6) \sim 5 \cdot 10^{-3}$ . Thus, we put  $a_{\max} = a(295 \text{ K}) = 0.705$  and  $a(288 \text{ K}) = 0.70$ . The characteristic width of the Gaussian curve is specified by the average value of modern climate sensitivity to albedo,  $\lambda_A \equiv -F_s(T_s) a'(T_s) \sim -0.6 \text{ W m}^{-2} \text{ K}^{-1}$  (Dickinson, 1985) at  $T_s = 288 \text{ K}$ . Finally, we take  $a_{\min} = 0.3$  for the asymptotic values of the Gaussian curve at large and low temperatures. We thus arrive at the following expression for  $a(T)$ , Fig. 2:

$$a(T) = 0.30 + 0.405 e^{-\left(\frac{T-295 \text{ K}}{60 \text{ K}}\right)^2} \quad (3.12)$$

Formulas (3.11) and (3.12) allow for a unambiguous derivation of the potential function  $U(T)$  (3.1), Fig. 3. Integration constant in Fig. 3 is chosen such that the value of  $U(T)$  in the point of the right extreme,  $T = 652 \text{ K}$ , is equal to zero.

As is clear from Fig. 3, there are only two physically stable states of the Earth's climate. These are the state of complete glaciation of the Earth's surface **1** and complete evaporation of the hydrosphere **2**. The intermediate stationary state **2** which corresponds to the maximum of the potential function  $U(T)$  is physically unstable.

The right ordinate axis in Fig. 3 shows  $U(T)$  scaled by the value of the global mean flux of the absorbed solar radiation  $F_e = 342 \text{ W m}^{-2}$ . In terms of  $F_e$ , the depth of the potential pit **3** corresponding to the stable state of complete evaporation of the hydrosphere, is equal to 58 K, while the depth of the potential pit **1** corresponding to complete glaciation is equal to 1.3 K. This value coincide by the order of magnitude with the depth of the glaciation po-

tential pit obtained by North et al. (1981), where it constituted  $\sim 4$  K. Besides the differences in the transmissivity functions  $b(T)$  employed, the difference in the depth of pits is explained by a larger value of climate sensitivity  $\lambda_A = -0.8$   $\text{W m}^{-2} \text{K}^{-1}$  used by North et al. (1981) as compared to the average value of  $\lambda_A = -0.6$   $\text{W m}^{-2} \text{K}^{-1}$  (Dickinson, 1985) accepted by us. The locations of glaciation pits approximately coincide,  $-37^\circ\text{C}$  and  $-43^\circ\text{C}$  in the work of North et al. (1981) and the present paper, respectively.

The stable state **1** of complete glaciation of the Earth's surface was previously investigated under the assumption of a linear dependence of  $F_{\text{out}}$  (3.1) on temperature in the whole interval of temperatures considered (Ghil, 1976; North, Coakley, 1979; North et al., 1981). The stability of this state was discussed in the context of possible changes in the solar constant  $I$ . The stable state **3** of complete evaporation of the hydrosphere arises due to the fact that the hydrosphere has a finite mass and the transmissivity function  $b(T)$  is therefore limited from below by  $b = b_{\text{min}} > 0$ . In the absence of this limitation the greenhouse effect and the surface temperature might increase infinitely. This phenomenon is called “runaway” greenhouse effect and was also extensively discussed in the literature (Ingersoll, 1969; Rasool, de Berg, 1970; Nakajima et al., 1992; Weaver, Ramanathan, 1995).

In the state of complete evaporation of the hydrosphere, the surface temperature ( $\sim 700$  K) and atmospheric pressure ( $\sim 300$  bars) are such that the atmospheric water finds itself above the critical point, where the difference between gas and liquid disappears. The major constituent of the atmosphere of Venus,  $\text{CO}_2$ , is also above the critical point there. Another feature of dense atmospheres is that the incoming solar radiation is absorbed predominantly in the upper atmospheric layers, with little reaching the planet's surface (Rossow, 1985). Both these features are approximately taken into account by setting the limiting value of the transmissivity function  $b_{\text{min}}$  equal to the corresponding value on Venus. We do not aim at exact determination of the stationary value of global mean surface temperature in state 3. We only assert that this stable state exists, similar to what is found on Venus.

Stable states 1 and 3 arise due to the practical constancy of the transmissivity function  $b$  and the coalbedo  $a$  in the considered intervals of high and low temperatures, see (3.3) and Figs. 2, 3. This, in its turn, is

caused by the constancy in the aggregate phase of the major greenhouse constituent — the hydrosphere is solid in state 1 and gaseous in state 3. The two minima of the potential function, Fig. 3a, can be joined by a continuous curve only via an unstable maximum in the temperature interval corresponding to the liquid hydrosphere. There are no physical reasons for appearance of a third minimum, which would be inevitably accompanied by two additional maxima in Fig. 3. Formation of such structures would have pointed to the existences of singularities in the temperature-dependent behaviour of  $a$  and  $b$  in the vicinity of the modern global mean surface temperature. These singularities should have had a clear physical interpretation, just as do the stable minima 1 and 3.

Transition from the state 1 to state 3 is accompanied by nearly a hundredfold monotonous decrease of  $b(T)$ , as compared to no more than a threefold change of coalbedo  $a(T)$ . Thus, despite that the exact temperature-dependent behaviour of coalbedo remains to a large extent unknown, no physically plausible assumptions about  $a(T)$  may significantly change the behaviour of  $U(T)$ , Fig. 3, leading to appearance of singularities of  $U(T)$  in the vicinity of  $T = 15^\circ\text{C}$ .

We determined our single parameter, constant  $\varepsilon$  in (3.10), demanding that the modern global mean surface temperature is stationary. In a stationary unstable state, where the curves  $F_{\text{in}}(T)$  and  $F_{\text{out}}(T)$  intersect, see (3.1), the temperature derivative of  $F_{\text{in}}(T)$  is larger than that of  $F_{\text{out}}(T)$ , see Fig. 4. A characteristic feature of such a state is that if  $F_{\text{in}}(T)$  were to decrease, the unstable stationary temperature would become larger, and vice versa. A colder stationary unstable climate arises at larger global mean values of the absorbed solar radiation  $F_{\text{in}} = Ia/4$ . If function  $a(T)$  is held unchanged, it corresponds to an upward shift of the curve  $F_{\text{in}}(T)$ , Fig. 4, without altering its form. At large values of  $a$  or  $I$ , the curve  $F_{\text{out}}(T)$  may remain below the curve  $F_{\text{in}}(T)$  at all  $T$  excluding very large ones. The stationary intersection points 1 and 2 will then disappear, and the only stationary stable state will be the gaseous hydrosphere, 3, which corresponds to the runaway greenhouse effect.

Generally, functions  $a(T)$ ,  $b(T)$  and  $F_{\text{out}}(T) \equiv \sigma T^4 b(T)$ , constructed on the basis of well-established physical laws for a given type of planetary surface (in our case — ocean), should be valid for description of all local areas of the same surface type. These areas differ from each other by the annual values of the incoming solar flux  $I_a$  (3.1). At the modern global mean surface temperature,  $T_s = 288\text{ K}$ ,  $F_{\text{out}}(T)$  approaches maximum, Fig. 4. It follows that in the equatorial oceanic regions, where the incoming solar flux is considerably larger than the global average, the regional value of  $F_{\text{in}}(T)$  would be larger than the maximum possible value of  $F_{\text{out}}(T)$ . The unstable stationary state will disappear, driving the equatorial regions to a state of runaway greenhouse. Thus, even if a given value of the global mean surface

temperature corresponds to a stationary unstable state, as determined by  $a(T)$  and  $b(T)$ , see (3.3), this does not by itself guarantee unstable stationarity for all local areas of the Earth's surface.

#### 4. Stability of the modern climate

The existence of life during the last several billion years, together with other paleodata (Savin, 1977; Watts, 1982; Berggren, Van Couvering, 1986), indicates that the modern Earth's climate is stable. It means that in the vicinity of the modern mean global surface temperature, the behaviour of  $a(T)$  and  $b(T)$  differs from (3.11), (3.12).

A stable state arises when in the point where  $F_{\text{out}}(T)$  and  $F_{\text{in}}(T)$  intersect, the temperature derivative of  $F_{\text{out}}(T)$  is larger than that of  $F_{\text{in}}(T)$ . In particular, extreme of  $U(T)$ , which is nearest to the absolute zero, will be always stable, see Figs. 4 and 5. Indeed,  $F_{\text{out}}(0) = 0$ , while  $F_{\text{in}}(0) > 0$ . Therefore,  $F_{\text{out}}(T)$  will cross  $F_{\text{in}}(T)$  from below, if only its slope is steeper than that of  $F_{\text{in}}(T)$ .

If the greenhouse effect is completely absent or temperature-independent ( $b(T) = \text{const}$ ), there is only one point of intersection between  $F_{\text{out}}(T)$  and  $F_{\text{in}}(T)$ , and it is stable. The second, unstable, point of intersection will arise, if the decrease in  $b(T)$  with temperature compensates growth proportional to  $T^4$ ,  $F_{\text{out}}(T) \equiv b(T)\sigma_R T^4$ , so that the derivative of  $F_{\text{out}}(T)$  becomes less than that of  $F_{\text{in}}(T)$ . Due to the existing physically transparent limitations  $0.3 \leq a(T) \leq 0.7$ , see (3.12), no changes in coalbedo  $a(T)$  are able compensate the growth of  $F_{\text{out}}(T) \propto \sigma T^4$  at  $b(T) = \text{const}$  and create a second point of intersection between  $F_{\text{out}}(T)$  and  $F_{\text{in}}(T)$ .

For a stable intersection point to appear in the vicinity of  $T \sim 15^\circ\text{C}$ , it is necessary that the curve  $F_{\text{out}}(T)$  makes here a zigzag, with three points of intersections with  $F_{\text{in}}(T)$ , of which two are unstable and one is stable, see Fig. 5b below. In the central part of this zigzag function  $b(T)$  is approximately constant, so that the curve  $F_{\text{out}}(T)$  crosses  $F_{\text{in}}(T)$  from below, just as in the absence of the greenhouse effect ( $b = 1$ ), but at higher values of temperature. Thus, generating the zigzag of  $F_{\text{out}}(T)$  and choosing the value of  $b < 1$ , it is possible to form a stationary stable state in the region of life-compatible temperatures. Within the central part of this zigzag, the derivative of  $F_{\text{out}}(T)$  is large than that of  $F_{\text{in}}(T)$ . The stable intersection point can therefore move to the right and to the left in response to regional changes in the incoming solar flux  $I_a$ . Thus, stable stationary states are formed in the regions with lower (polar) and higher (equatorial) temperatures as compared to the global average.

The temperature-dependent behaviour of  $b(T)$  in the vicinity of the

modern value of global mean surface temperature can be derived from observations. As shown in numerous studies (Budyko, 1969; North, Coakley, 1979; North et al., 1981; Raval, Ramanathan, 1989; Stephens, Greenwald, 1991a,b), the regional function  $b(T)$  remains is a linear function of temperature within a broad temperature interval. The gentle slope of this almost constant function ensures a stable stationary state of the modern climate for all physically plausible functions  $a(T)$ . As the mean value of a linear function coincides with its value of the mean argument,  $\overline{f(x)} = f(\bar{x})$ , one can assume that measurements of  $b(T)$  in different regions and at different temperatures describe the temperature-dependent behaviour of the global mean function  $b(T)$ .

The observed greenhouse effect dependence on temperature within the temperature interval from  $\sim 0^\circ\text{C}$  (273 K) to  $\sim 30^\circ\text{C}$  (303 K) was described by Raval and Ramanathan (1989) and Stephens and Greenwald (1991b) for the clear and cloudy sky, respectively. Quantitatively, the obtained results can be summarised as follows, Fig. 5a:

for  $273 \text{ K} \leq T \leq 299 \text{ K}$ :

$$b_{\text{RR}}(T) = 0.67 - 2.6 \cdot 10^{-3} \cdot (T - 288) \quad (5.1a)$$

$$b_{\text{SG}}(T) = 0.59 \quad (5.1b)$$

for  $299 \text{ K} \leq T \leq 303 \text{ K}$ :

$$b_{\text{RR}}(T) = 0.64 - 9.6 \cdot 10^{-3} \cdot (T - 299) \quad (5.2a)$$

$$b_{\text{SG}}(T) = 0.59 - 85.5 \cdot 10^{-3} \cdot (T - 299) \quad (5.2b)$$

Function  $b_{\text{RR}}(T)$  is obtained by linear approximation of the point measurements of  $B_{\text{RR}}(T) \equiv 1 - b_{\text{RR}}(T)$  presented in Fig. 2 of the work of Raval and Ramanathan (1989). The approximation was performed separately for temperature intervals  $273 \text{ K} \leq T \leq 299 \text{ K}$  and  $299 \text{ K} \leq T \leq 303 \text{ K}$ . The correlation coefficient equals 0.784 ( $\sim 1500$  d.f.) and 0.653 ( $\sim 500$  d.f.) for the resulting curves (5.1a) and (5.2a), respectively.

According to Stephens and Greenwald (1991b, p. 15334, Fig. 9a), the value  $1/b_{\text{SG}}(T)$  changes from 1.7 to 2.4 over the temperature interval from 275 K to 301 K, still remaining approximately equal to 1.7 at  $T = 299 \text{ K}$  and then rising rapidly up to 2.4 at  $T = 299 \text{ K}$ . Function  $b_{\text{SG}}(T)$  (5.1b)-(5.2b) reflects this behaviour.

As follows from comparison of (5.1) and (5.2), the observed temperature-dependent behaviour of transmissivity functions  $b_{\text{RR}}(T)$  and  $b_{\text{SG}}(T)$  differ significantly from the physical behaviour (3.11) in the interval from  $0^\circ\text{C}$  to  $26^\circ\text{C}$ , see (5.1). At  $T > 26^\circ\text{C}$  both functions undergo drastic changes (which was noted by both Raval and Ramanathan (1989) and Stephens and Greenwald (1991b)) and approach the physical behaviour (3.11) in the interval from  $26^\circ\text{C}$  to  $30^\circ\text{C}$ . The slope of  $b_{\text{RR}}(T)$  in this interval exactly coincides with that of (3.11). The slope of  $b_{\text{SG}}(T)$  is about ten times steeper, Fig. 5a.

As far as measurements RR were obtained for a clear sky, while

measurements SG — for cloudy sky only, curves  $b_{RR}(T)$  and  $b_{SG}(T)$ , shown in Fig. 5a, lie above and below the global mean value  $b(288 \text{ K}) = 0.61$ , respectively. The true empirical curve  $b_{\text{emp}}(T)$ , corresponding to mean cloudiness, goes between the curves  $b_{RR}(T)$  and  $b_{SG}(T)$ . Taking mean global cloudiness of about 50% (Rennó et al., 1994), we have

$$b_{\text{emp}}(T) = \frac{b_{RR}(T) + b_{SG}(T)}{2} = \begin{cases} 0.63 - 1.3 \cdot 10^{-3}(T - 288), & 273 \leq T \leq 299 \\ 0.62 - 47.6 \cdot 10^{-3}(T - 299), & 299 \leq T \leq 303 \end{cases} \quad (5.3)$$

We note that  $b_{\text{emp}}(288 \text{ K}) = 0.63$ , which only slightly differs from the global mean value of  $b(288) = 0.61$ .

At  $T = 273 \text{ K}$  function  $b_{\text{emp}}(T)$  (5.3) coincides with function  $b_{\text{NC}}(T)$ , which is constructed on the basis of a linear dependence of  $F_{\text{out}}$  on temperature used in the work of North and Coakley (1979) and North et al. (1981),  $F_{\text{out}}(T) \equiv b_{\text{NC}}(T)\sigma_R T^4 = [203.3 + 2.09(T - 273)] \text{ Bt m}^{-2}$ . However, at  $T = 234 \text{ K}$  ( $-39^\circ\text{C}$ ) function  $b_{\text{NC}}(T)$  thus defined starts to diminish with further decrease of temperature and becomes negative at  $T \leq 276 \text{ K}$  ( $-97^\circ\text{C}$ ). Such a behaviour is physically unjustified. With decreasing temperature the transmissivity function  $b(T)$  should increase monotonously, Fig. 1, governed by the diminishing amount of the major greenhouse substances (water and clouds) in the atmosphere. At  $T \rightarrow 0$  the linear dependence  $b_{RR}(T)$  also yields a physically meaningless value greater than unity. Constancy of  $b_{SG}(T)$  at  $T \rightarrow 0$  is physically implausible as well: as far as with decreasing temperature the atmospheric water content, including liquid water, is diminishing, the greenhouse effect of cloudy sky should diminish as well.

It follows that the real behaviour of  $b_{\text{emp}}(T)$  at certain  $T < 273 \text{ K}$  differs considerably from (5.3) and approaches the physically sensible asymptotic values described by (3.11), Fig. 1. The unknown behaviour of  $b_{\text{emp}}(T)$  at  $T < 273 \text{ K}$  is represented in Fig. 5a by the dashed line. Intersections of  $b_{\text{emp}}(T)$  with the physical function  $b(T)$  (3.11) occur at  $T = 266 \text{ K}$  and  $T = 302 \text{ K}$ . The slope of the model dashed line  $b_{\text{emp}}(T)$  at  $266 \text{ K} \leq T \leq 273 \text{ K}$  is chosen arbitrarily to equal one half of the slope of  $b_{\text{emp}}(T)$  at  $299 \text{ K} \leq T \leq 303 \text{ K}$  under the assumption that cloudiness at  $T \leq 273 \text{ K}$  constitutes one half of cloudiness at  $T \geq 299 \text{ K}$ .

It is important to note that when the behaviour of the regional functions  $b(T)$  deviates from the linearity observed in the vicinity of the mean global temperature, cf. (5.1) and (5.2), the global function  $b(T)$  will be better described by those regional functions that correspond to the largest areas of the Earth's surface. For example, the modern regional function  $b(T)$  corresponding to low latitudes, i.e. extensive equatorial territories with high temperatures ( $T > 20^\circ\text{C}$ ), should describe the behaviour of global function  $b(T)$  at high temperatures better than the modern high-latitude regional function  $b(T)$ , which correspond to limited polar regions with low temperatures ( $T < -20^\circ\text{C}$ )

may describe the behaviour of the global function  $b(T)$  at low global mean surface temperatures. Thus, it may well be the case that the behaviour of the global function  $b_{\text{emp}}(T)$  at low temperatures (dashed line in Fig. 5) cannot be predicted on the basis of the modern regional functions  $b(T)$  for low latitudes, making the palaeoclimatic data the only source of information for the corresponding temperature interval.

In Fig. 6 we show the potential function  $U_{\text{emp}}(T)$ , obtained from (3.1) using the transmissivity function  $\tilde{b}(T)$ , which is constructed by merging  $b_{\text{emp}}(T)$  and  $b(T)$  (3.11):

$$-\frac{dU_{\text{emp}}(T)}{dT} \equiv \tilde{b}(T)\sigma T^4 - a(T), \quad (5.4)$$

$$\tilde{b}(T) = \begin{cases} b(T), & T \leq 266 \text{ K} \\ b_{\text{emp}}(T), & 266 \text{ K} \leq T \leq 302 \text{ K} \\ b(T), & T \geq 302 \text{ K} \end{cases}$$

The coalbedo  $a(T)$  is given by (3.12). The integration constant in (5.4) is chosen so that the position of the left extreme of  $U_{\text{emp}}(T)$  (complete glaciation) coincides with that of the physical potential function  $U(T)$ , Fig. 3.

As is clear from Fig. 6, the stationary state of the modern climate corresponds to a potential pit surrounded by potential barriers. The minimum of the potential pit corresponds to the stationary stable state 2. The maxima of the surrounding potential barriers corresponds to unstable stationary states, where the probabilities of transition to states 1 and 2 from the left barrier and to states 2 or 3 from the right barrier are equal.

## 5. Conclusions: The nature of modern climate stability

Solar energy supports all dynamic processes on the Earth's surface, including life. The planet absorbs the maximum possible amount of solar energy when the coalbedo is at its maximum (minimal albedo). The stationary state of the modern climate corresponds to coalbedo close to its maximum value possible under terrestrial conditions,  $a \approx 0.7$ . In the absence of the greenhouse effect, i.e. at  $b = 1$ , the global mean surface temperature would be equal to the effective temperature of the outgoing long-wave radiation, i.e.  $T = T_e = 255 \text{ K}$  ( $-18 \text{ }^\circ\text{C}$ ), making questionable the possibility of life existence. The observed increase of the global mean surface temperature up to the modern optimal for life values ( $T_s = +15 \text{ }^\circ\text{C}$ ) is only possible due the non-zero greenhouse effect, which corresponds to  $b = 0.6$ , see (3.2). The greenhouse effect is generated by the necessary atmospheric concentrations of greenhouse substances that constitute fractions of per cent of the total atmospheric mass.

The decrease of  $b$  from unity to the optimal for life value of 0.6 might retain the physical stability of the stationary surface temperature if only the greenhouse effect is ensured by greenhouse substances with concentrations independent of or only slowly dependent on temperature, like, e.g.  $\text{CO}_2$ . This will make  $b$  practically constant in the vicinity of the stationary temperature. However, the major absorption band of  $\text{CO}_2$  traps only 19% of the Earth's thermal radiation at  $T = 288$  K, see (3.5). Even if  $\text{CO}_2$  concentration is infinitely increased, which corresponds to disappearance of the second item in sum (3.4), the value of  $b$  can only be diminished to 0.81. Thus, to reach the needed  $b = 0.6$  it is necessary to involve atmospheric water vapour and cloudiness.

However, in the presence of liquid water on the planet's surface, which is also favourable for life, there appears a physical positive feedback between the amount of water vapour and clouds in the atmospheric column (and, consequently, their optical thicknesses  $\tau_s$ ) and the surface temperature, see (3.9) and (3.10). With an account made for the non-radiative fluxes of thermal energy (convection and latent heat), this positive feedback makes the stationary surface temperature  $T_s = 288$  K physically unstable.

To ensure stability at the same value of surface temperature some controlling processes should be active in the vicinity of this temperature, which will change the basic physical temperature dependence of optical thickness of water vapour and cloudiness.

Differences in the incoming solar fluxes at different latitudes lead to the fact that the equatorial and polar regions are characterised by higher and lower temperatures as compared to the global average, respectively. Processes of global circulation, that arise due to these temperature differences, work to diminish them. An account of global circulation processes is made in the three-dimensional climate models. If the Earth's surface were entirely flat and perpendicular to the plane of its orbit, all regions would receive equal amounts of solar radiation and the processes of global circulation ceased to exist. It is unlikely that the modern climate stability is due to global circulation processes, i.e. exclusively due to the geometry of the Earth's surface.

Global circulation processes are unable to change the dependence of the greenhouse effect and transmissivity function  $b(T)$  on local values of temperature. As discussed above, these local dependencies may cause exponential runaway greenhouse effect in high latitudes, which will contribute to global instability. Moreover, it is unlikely that the global circulation processes are responsible for the drastic increase of the outgoing radiation with decreasing temperature, which takes place at the left and right parts of the zigzag  $F_{\text{out}}(T)$ , Fig. 5b and ensures the possibility of modern stationary surface temperature being stable. Within the observed right part of the zigzag  $F_{\text{out}}$  changes by more than  $60 \text{ W m}^{-2}$  within the temperature interval from 299 K to 302 K. This

corresponds to 15% of the global mean flux of thermal radiation at the Earth's surface,  $F_s$ . In comparison, the average power of global circulation constitutes no more than 2.5% of that value (Kellog, Schneider, 1974; Peixoto, Oort, 1984; Chahine, 1992).

We can conceive only one possible explanation of the observed climate stability, namely the existence of a biotic control of the greenhouse effect (Gorshkov et al., 2000). Without aiming to prove this statement here and considering it as a hypothesis, we list here the major arguments in its support.

Modern concentrations of all the major greenhouse substances — water vapour, clouds and  $\text{CO}_2$  — are under control of the global biota of Earth, being involved into global biogeochemical cycles. Terrestrial vegetation determines the rate of water evaporation from land through regulation of transpiration — release of water vapour from plant leaves. In the ocean, the marine biota produces surface-active substances that considerably impact the rate of water evaporation from the ocean surface. These essentially non-random processes may change the dependence of relative humidity on temperature, thus changing coefficient  $\varepsilon$  in (3.10) and making it temperature dependent. Changing concentrations of biological aerosols that serve as condensation nuclei in the atmosphere, the biota is able to change ratio  $r$  (3.9) between optical thicknesses of water vapour and liquid water.

In the ocean, the dominant parameter controlling absorption of the incident solar radiation is the concentration of photosynthetic pigment contained in phytoplankton cells (Sathyendranath, 1991). Regulating this parameter, the marine biota is able to change the average depth at which the short-wave radiation is absorbed and dissipated into heat. The resulting heat flux will thus transfer from different depths, covering different numbers of effective water layers  $m_w$ . In the atmosphere, the optical thickness of water vapour is exponentially dependent on temperature, see (3.10). In contrast,  $m_w$  is uniquely determined by the average depth where solar radiation is absorbed, which, in its turn, is dependent on the concentrations of biotically controlled substances. As far as in the stationary case thermal radiation into space is fixed by the value of coalbedo, see (3.2), it is possible, by increasing penetration of sunlight into depth, to move a considerable or even dominant part of the greenhouse effect into the ocean. Temperature of the oceanic surface corresponding to  $m_w = 0$  will then decrease, an effect similar to the decrease of temperature of the upper atmospheric layers corresponding to small values of optical depth  $\tau$ . Accordingly, the atmospheric contribution into the greenhouse effect will drop exponentially. It is not unlikely that the well-known transparency of equatorial waters may be a manifestation of such biotic cooling of the areas with largest incoming solar fluxes. On the contrary, in the subpolar regions the biota may increase the turbidity of surface waters, thus making sunlight be absorbed near the surface and generating the full possible green-

house effect in the atmosphere due to the increase of surface temperature up to the maximum possible value.

The atmospheric concentration of CO<sub>2</sub>, the second important greenhouse gas, is also under biotic control, and there is. Given the large number of publications on this topic, here we only note that, given the global biological production (and, consequently, decomposition) being of the order of 10<sup>2</sup> Gt C and the global atmospheric CO<sub>2</sub> content of the order of 10<sup>3</sup> Gt (Gorshkov et al., 2000), even a minor imbalance in the fluxes of biological production and decomposition may lead to drastic changes of atmospheric CO<sub>2</sub> content over geologically instantaneous time periods. E.g. if biological decomposition exceeds biological production by only 10%, the CO<sub>2</sub> concentration will double in less than 100 years. No geophysical processes are comparable in power with the biological control of atmospheric CO<sub>2</sub>.

We note finally that the biological processes of climate control are based on consumption of solar energy. Accordingly, the maximum (Carnot) efficiency  $\eta_B$  of these processes is equal to  $\eta_B = (T_{\text{Sun}} - T_{\text{Earth}})/T_{\text{Sun}} \approx (6000 \text{ K} - 300 \text{ K})/6000 \text{ K} \approx 0.95$ . In the meantime, the maximum efficiency  $\eta_{GC}$  of the processes of global circulation, which is based on the difference between the temperatures of the polar and equatorial regions, is equal to  $\eta_{GC} = (T_{\text{equator}} - T_{\text{poles}})/T_{\text{Earth}} \approx 30 \text{ K}/300 \text{ K} \approx 0.1$ , which is an order of magnitude lower than  $\eta_B$ . Thus, also from this point of view, the biotic potential for climate control is larger than that of physical dynamic processes on the Earth's surface.

In the modern conditions of increasing anthropogenic impact of the global environment, including the biota, further investigations into the nature of climate stability on Earth are undoubtedly needed.

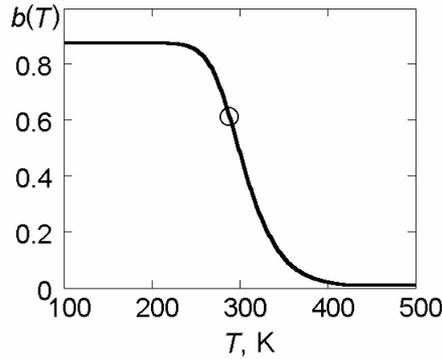


Fig. 1. Temperature dependence (3.11) of the theoretical physical transmissivity function  $b(T)$ . The modern global value  $b(288 \text{ K}) = 0.61$  is shown by the empty circle.

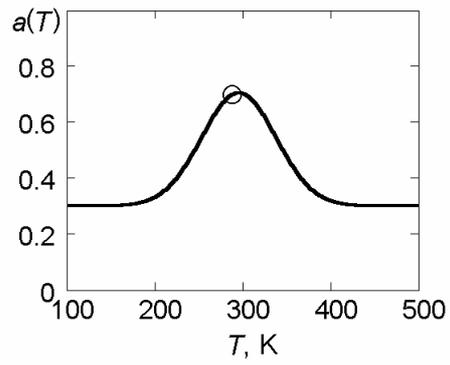
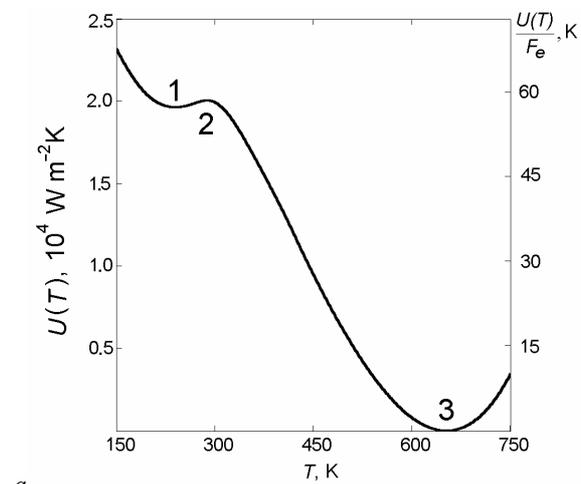
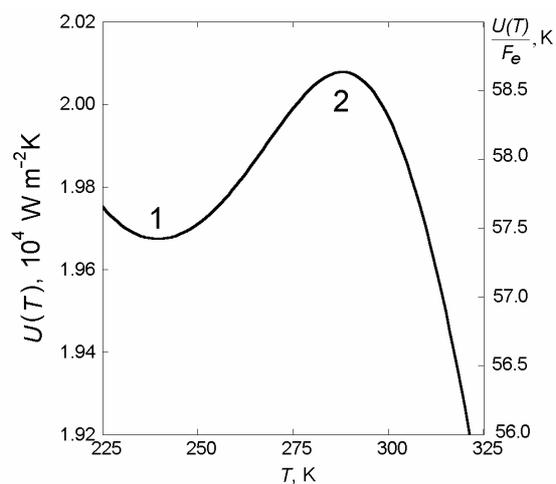


Fig. 2. Temperature dependence (3.12) of the coalbedo  $a(T)$ . The modern global value  $a(288 \text{ K}) = 0.70$  is shown by the empty circle.



*a*



*b*

Fig. 3a,b. Theoretical physical potential function  $U(T)$  (3.1) constructed on the basis of  $b(T)$  (3.11) and  $a(T)$  (3.12), see Fig. 1 and 2. States 1 and 3 are stable, corresponding to complete glaciation of the planet and complete evaporation of the hydrosphere, respectively. State 2, corresponding to the modern global mean surface temperature, is unstable.

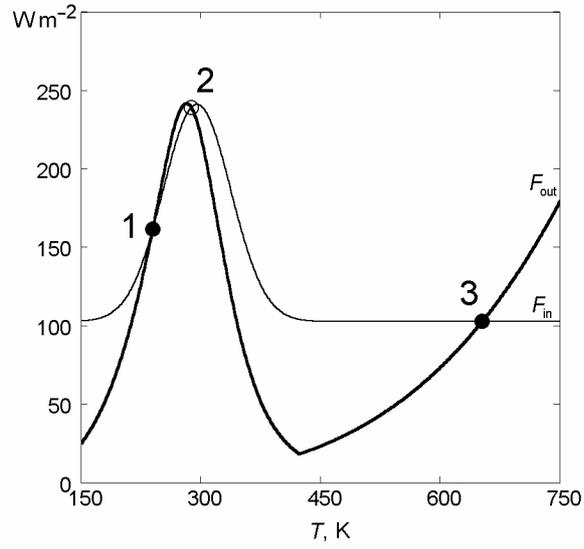


Fig. 4. Temperature dependences of the theoretical physical global mean fluxes  $F_{in}(T)$  and  $F_{out}(T)$  of the absorbed short-wave (thin curve) and emitted by the planet into space long-wave (thick curve) radiation, see (3.1). The stationary stable points of intersection  $F_{in}(T) = F_{out}(T)$  are marked with filled circles. The empty circle shows the unstable stationary state. See legend to Fig. 3 for designations of numbers.

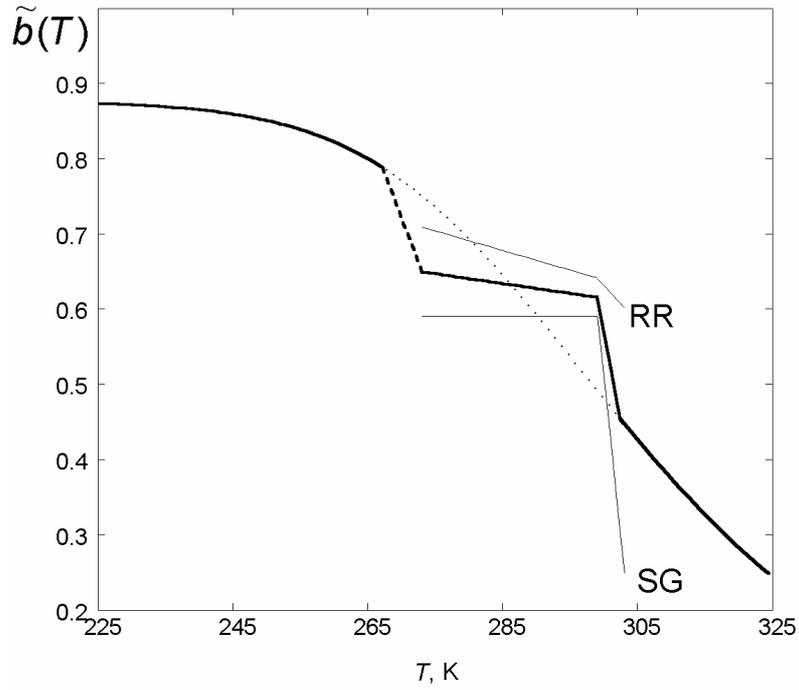


Fig. 5a. The observed transmissivity function  $b_{\text{emp}}(T)$  (5.3). RR — function  $b_{\text{RR}}(T)$  for clear sky, see (5.1a), (5.2a); SG — function  $b_{\text{SG}}(T)$  for the cloudy sky, see (5.1b), (5.2b). Thick solid curve interrupted by the dashed line — function  $\tilde{b}(T)$ , see (5.3) and (5.4). The dashed line corresponds to the tentative temperature interval where the behaviour of  $b_{\text{emp}}(T)$  remains unknown. Dotted curve — theoretical function  $b(T)$  (3.11) in the temperature interval  $266 \text{ K} \leq T \leq 302 \text{ K}$ , cf. Fig. 1.

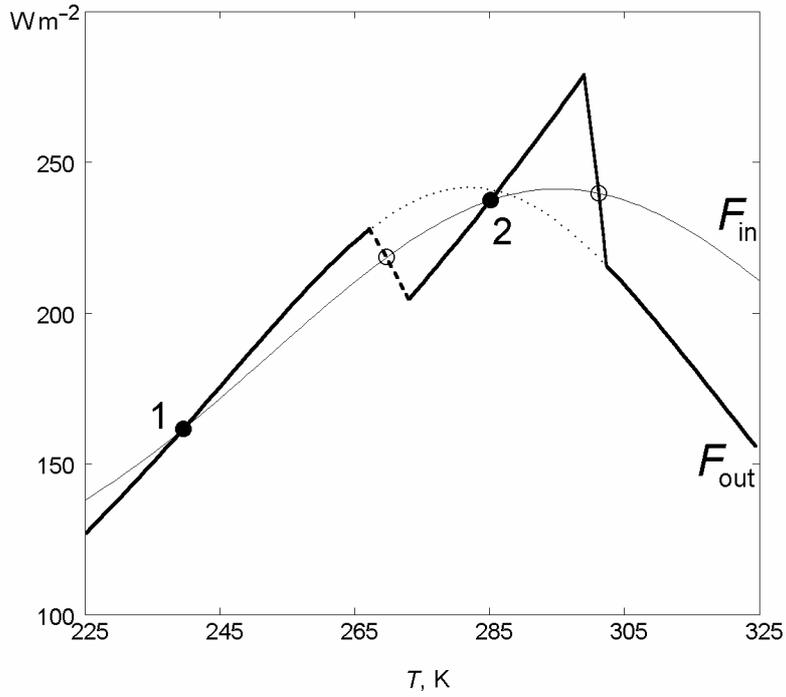


Fig. 5b. Temperature dependence of the flux of long-wave radiation emitted by the planet into space,  $F_{\text{out}}(T)$ , taking into account its observed behaviour in the vicinity of modern global mean surface temperature. Thick solid curve interrupted by dashed line —  $F_{\text{out}}(T) \equiv \tilde{b}(T)\sigma T^4$ ,  $\tilde{b}(T)$  is shown in Fig. 5a. Dotted curve — theoretical physical behaviour of  $F_{\text{out}}$ , see Fig. 4. Thin line — the global mean flux of the absorbed solar radiation  $F_{\text{in}}(T)$ , see Fig. 4. The stationary stable points of intersection  $F_{\text{in}}(T) = F_{\text{out}}(T)$ , corresponding to complete glaciation (1) and modern climate (2) are marked with filled circles. The empty circle shows the unstable stationary states corresponding to the potential stability barriers of the modern climate, see Fig. 6.

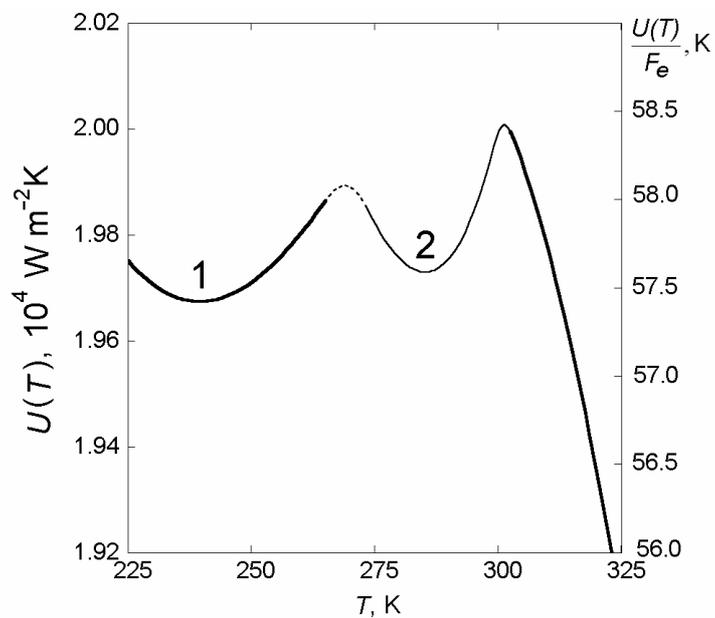


Fig. 6. The potential function  $U_{\text{emp}}(T)$  (5.3) of the modern Earth's climate, thick line. Thin line interrupted by dotted line — theoretical physical function  $U(T)$ , see Fig. 3b. Dashed line, as in Fig. 5a,b, corresponds to the unknown behaviour of  $b_{\text{emp}}(T)$ , see (5.3) and text.

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