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Empirical evidence for the condensational theory of hurricanes

A.M. Makarieva, V.G. Gorshkov, A.V. Nefiodov *

Theoretical Physics Division, National Research Centre "Kurchatov Institute", B.P. Konstantinov Petersburg Nuclear Physics Institute, 188300 Gatchina, St. Petersburg, Russia

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ABSTRACT

It is shown that recent data on gravitational power of precipitation in hurricanes agree with the theoretical relationships previously obtained for the dynamic power of hurricanes driven by condensation of atmospheric water vapor. Total power of the hurricane derived from water vapor condensation is equal to the sum of wind power and gravitational power of precipitation, with the ratio between the two governed by the ratio of the atmospheric scale height for water vapor and the precipitation path length.

1. Introduction

Predicting the intensity of hurricanes (tropical cyclones) is a major challenge for atmospheric science [1]. Using the approach that considers heat exchange with the ocean as hurricane driver it has not been possible to predict either the trend or the mean value of hurricane intensity in recent decades (see Fig. 2b in work [2]). According to the recently developed condensational theory, hurricane energetics is determined by condensation of water vapor that has accumulated in the atmosphere during a long period preceding hurricane formation [3]. Condensation of water vapor results in a positive difference between the power associated with the vertical gradient of the partial pressure p_v of water vapor, $-\mathbf{w}\nabla p_v$, and the power of gravity acting on the ascending water vapor with mass density ρ_v, $-\mathbf{wg}\rho_v$, where $\mathbf{g}$ is the acceleration due to gravity and $\mathbf{w}$ is vertical air velocity. This difference, $-\mathbf{w}\nabla p_v + \mathbf{wg}\rho_v > 0$, generates the power of horizontal wind $-\mathbf{u}\nabla p$, where $\mathbf{u}$ is horizontal air velocity and p is air pressure. Thus, power related to the vertical gradient of water vapor partial pressure, $-\mathbf{w}\nabla p_v$, comprises both the power of kinetic energy generation and the gravitational power of precipitation. The latter is equal to the power at which condensing water vapor is lifted in the gravitational field. This allows one to relate the power of precipitation and kinetic energy generation. Recently published empirical data on precipitation in hurricanes [2] permit verification of the obtained theoretical relationship. In this paper, it is shown that the available observational data are in agreement with the condensational theory.

* Corresponding author.
E-mail address: anef@thd.pnpi.spb.ru (A.V. Nefiodov).

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2. Condensational power

The power of condensation per unit air volume, s (W m^{-3}), and molar condensation rate, σ (mol m^{-3} s^{-1}), are equal to $s \equiv -wp\partial\gamma/\partial z$ and $\sigma \equiv -wN\partial\gamma/\partial z$, where $p = NRT$ is moist air pressure, N (mol m^{-3}) is molar density, $R = 8.31$ J mol^{-1} K^{-1} is the universal gas constant, T is absolute temperature, z is height above the sea level, $\gamma \equiv p_v/p$ is the relative partial pressure of water vapor [3]. Power s does not depend on the difference in molar masses of dry air and water vapor due to the fact that all air constituents move at the same velocity [4,5].

Condensational power goes to generate kinetic energy: $s = u\partial p/\partial r$, where u is radial velocity and r is radius counted from the center of condensation area. Gravitational power of precipitation s_P is equal to the power at which gravitational potential energy is released in water precipitating from height z: $s_P = \sigma M_v g z \equiv sz/h_v$, where $M_v = 18$ g mol^{-1} is molar mass of water vapor, $h_v \equiv RT/M_v g$ is the exponential scale height determined by the ideal gas equation of state for water vapor, $p_v = N_v RT = \rho_v g h_v$, where $N_v = \rho_v/M_v$ is molar density of water vapor [6].

These powers expressed per unit surface area of the atmospheric column, where condensation occurs, and denoted below with W (W m^{-2}) become

$$W \equiv \int_{z_1}^{z_2} s dz = wp[\gamma(z_1) - \gamma(z_2)] = w\Delta p, \tag{1}$$

$$\Delta p \equiv \beta p_v(z_1), \quad \beta \equiv \frac{\gamma(z_1) - \gamma(z_2)}{\gamma(z_1)}. \tag{2}$$

From the equality $s = u\partial p/\partial r$ we have

$$W = \int_{z_1}^{z_2} u\frac{\partial p}{\partial r}dz = h_u u(z_1)\left.\frac{\partial p}{\partial r}\right|_{z_1}, \tag{3}$$

$$h_u \equiv \frac{1}{u(z_1)}\int_{z_1}^{z_2} u(z)dz, \tag{4}$$

$$w = \frac{h_u}{r}\frac{\partial ur}{\partial r}. \tag{5}$$

In Eq. (4), z_2 and hence h_u can slightly depend on r. For the gravitational power of precipitation W_P we have

$$W_P = \int_{z_1}^{z_2} s\frac{z}{h_v}dz = W\frac{H_P}{h_v}, \tag{6}$$

$$H_P \equiv \frac{1}{\beta\gamma(z_1)}\int_{z_2}^{z_1} z\frac{\partial\gamma}{\partial z}dz. \tag{7}$$

Here z_1 and z_2 are the lower and the upper heights of the layer where condensation occurs ($z_1 < z_2$), β is degree of completeness of condensation [3], and H_P is the mean precipitation path length [6]. In the definition of h_u (4) it is taken into account that air pressure changes over z significantly more slowly than does u. Continuity equation (5) is valid for $\gamma \ll 1$. The kinetic power of hurricane is equal to $W = PRT$, where $P = w\beta N_v$ ($\text{mol}\,H_2O\ \text{m}^{-2}\,\text{s}^{-1}$) is precipitation [3]. Values of W (3) and W_P (6) can be derived from observations. Note that gravity (constant g) enters only the expression for the gravitational power of precipitation W_P, while the kinetic power W is independent of g.

Using continuity equation (5), we can write Euler equation and Bernoulli integral for the dependence of air pressure (3) on radius r:

$$-\frac{\partial p}{\partial r} = -\Delta p\frac{\partial \ln ur}{\partial r} = \frac{\rho}{2}\frac{\partial V^2}{\partial r}, \tag{8}$$

$$p - p_1 = \Delta p \ln\frac{ur}{u_1 r_1} = \frac{\rho}{2}\left(V_1^2 - V^2\right), \tag{9}$$

where ρ is air density at $z = z_1$, V is total air velocity, $V^2 = u^2 + v^2 + w^2$, v is tangential velocity of rotation around the condensation center, $p_1 \equiv p(r_1)$, $u_1 \equiv u(r_1)$, $V_1 \equiv V(r_1)$ are pressure and velocities at the periphery of the hurricane, r_1 is outer border of condensation area. In Eq. (9), the potential energy of condensation Δp is enhanced by the logarithmic term in that grows with decreasing r. This reflects, according to the Bernoulli equation, the narrowing of the air flow as it approaches the windwall. Turbulent friction becomes essential only in the vicinity of the windwall, where the hurricane eye forms, and the height of the condensation layer $z_2 - z_1$, which determines H_P (7), reaches its maximum.

3. Energy and radius r_0 of the hurricane eye and windwall

At the outer border of the hurricane ($r = r_1$), angular momentum a is equal to $a = \omega r_1^2 = v_1 r_1$, where $v_1 = \omega r_1$, $\omega = \Omega\sin\varphi$, $\Omega = 7.3 \cdot 10^{-5}\ \text{s}^{-1}$ is the angular frequency of Earth's rotation, φ is the meridional angle counted from the equator. The angular momentum consists of two parts. The first part is a constant magnitude a_{in}, which does not depend on r, and the second part is a magnitude proportional to ur: $a = a_{in} + a_{out}ur/u_1 r_1$ [3]. As the air flows towards the windwall in the boundary layer $z < z_1$ (where condensation does not occur), tangential velocity grows governed by momentum conservation as $v = a_{in}/r$. It reaches maximum value of $v_0 = a_{in}/r_0$ in the windwall.

Therefore, Euler equation (8) and Bernoulli integral (9) determine $u(r_0) \equiv u_0 = u_1$ at $r = r_0$. For $r < r_0$ equation (9) does not have real solutions [7]. As follows from Eq. (9) at $r = r_0$ total velocity is equal to $V_0 = v_0$ [3]. Thus Eq. (9) determines the windwall and eye radius of hurricane, where all condensational energy is concentrated into the kinetic energy of the rotating windwall:

$$\frac{\rho}{2}\left(v_1^2 - v_0^2\right) = \Delta p \ln\frac{r_0}{r_1}, \tag{10}$$

where $v_0 = a_{in}/r_0$.

Using dimensionless variables Eq. (10) can be written in a compact way [3]:

$$\frac{a_{in}^2}{r_0^2} = a_{in}^2 - \ln r_0, \quad v_0 = \frac{a_{in}}{r_0}. \tag{11}$$

4. Wind power at the windwall

Hurricane eye is a vortex in approximate cyclostrophic balance: the centrifugal force acting on the rotating air is compensated by the centripetal force of the pressure gradient that arises as the air is removed from the eye during its formation. The pressure gradient is perpendicular to rotation velocity and does not generate work and power. The eye is a form of turbulent resistance to the formation of the windwall: the eye reduces the kinetic energy of the windwall by $(r_0/r_e)^2 = 1.65^{-2}$, that is, by 37%, where $r_e - r_0$ is the radial extension of the windwall and r_e determines approximately its external radius (see work [3] for details).

If the hurricane and the windwall did not move as a whole, the windwall would exist for a short period of the order of $r_1/\overline{u}$ two days. It would represent a single act of conversion of potential energy p_v of locally accumulated water vapor into the finite kinetic energy of the rotating windwall and eye. Hurricane power would not be stationary and differed from zero during the period of windwall formation only.

Hurricane is not a closed stationary cycle. The condensing water vapor and hurricane air that rise in the windwall do not return to the windwall, but leave the hurricane area. When all water vapor is used up and condensation ceases, air motion outside the hurricane area does not influence hurricane energetics. Stationary hurricane power is only possible, if there is a continuous inflow of the potential energy p_v of water vapor. This occurs, when the hurricane moves along to new areas with previously accumulated water vapor that has not yet been used up. The hurricane can be compared to a forest fire or to an animal moving over its feeding territory and consuming the available food [8].

As the hurricane moves with the so-called translation or drift velocity $U \approx 10\ \text{m}\,\text{s}^{-1}$, there is a flux of water vapor via the vertical cross section of the entire hurricane, which is equal to $2r_1 h_u U$. In the rest frame of the hurricane, this flux corresponds to flux of water vapor via the cylindrical outer wall of the hurricane, $2\pi r_1 h_u u_1$. Thus the radial velocity at the outer border and at the windwall are related to translation velocity as $u_1 = u_0 = U/\pi$ [3,7].

Taking into account that within the relatively narrow windwall the velocity u_0 is relatively constant, $\partial u_0/\partial r \ll u_0/r_0$ [3], we can write (5) as

$$w_0 = \frac{h_u}{r_0}u_0 = \frac{h_u}{\alpha r_1}\frac{u_0}{v_1}v_0, \quad a_{in} = v_0 r_0 \equiv \alpha v_1 r_1. \tag{12}$$

As a result, for the wind power W_0 (1) and for the gravitational power of precipitation W_{P0} (6) at the windwall we obtain

$$W_0 = A_0 v_0, \quad A_0 \equiv \Delta p\frac{h_u}{\alpha r_1}\frac{u_0}{v_1}, \tag{13}$$

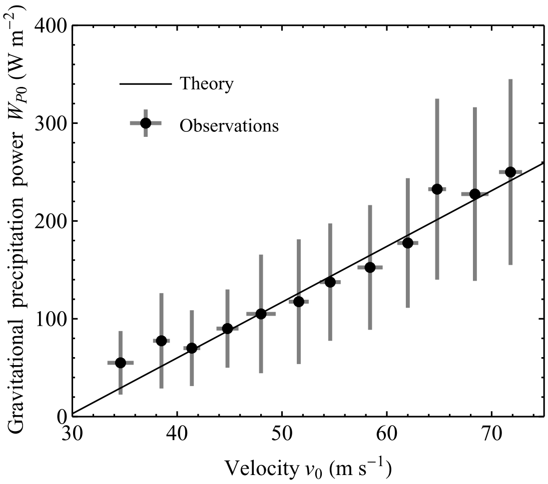

Fig. 1. Dependence of gravitational precipitation power in the windwall with respect to hurricane velocity. Empirical data (circles with gray bars showing two standard deviations in each bin) adopted from Fig. 1d of work [2] and theoretical curve (19) (black solid line).

$$W_{P0} = A_0 \frac{H_{P0}}{h_v} v_0. \tag{14}$$

Equations (13) and (14) are consistent with empirical observations analyzed in work [2]. Parameters involved into these equations take the following characteristic values: $r_1 = 300$ km [2], $h_u = 4.5$ km, $v_1 = \omega r_1 = 7.5$ m s^{-1}, $\omega = \Omega \sin\varphi = 2.5 \cdot 10^{-5}$ s^{-1}, $\varphi = 20°$, $u_0 = U/\pi = 3.2$ m s^{-1}, $U = 10$ m s^{-1}, $\gamma = 3 \cdot 10^{-2}$, $\beta = 0.4$, $\alpha = 0.4$, $p = 10^5$ Pa, $\Delta p = \beta\gamma p = 12$ hPa [3], $H_{P0} = 4$ km [6], $h_v = 14$ km ($T = 300$ K), $W_{P0}/W_0 = H_{P0}/h_v = 0.29$. It yields $A_0 H_{P0}/h_v = 5.6$ J m^{-3}, that is in agreement with the linear term relating W_{P0} and v_0 derived from empirical data in Ref. [2] (see Fig. 1).

Relationships (13) and (14), where windwall power is proportional to v_0, are strictly valid, if only the power ~~per unit area~~ of the windwall is significantly greater than the ~~mean~~ power ~~per unit area of the atmospheric column~~ in the vortex as a whole. This condition holds in compact tornadoes, but does not hold in hurricanes, where the windwall power ~~characterizes the mean hurricane power as well~~. Thus velocity v_0 is referred in the literature as "hurricane intensity".

With increasing angular momentum a_{in} the windwall and eye of the hurricane disorganize, the energy ceases to concentrate and the hurricane turns into an ordinary cyclone, where power is relatively evenly distributed over the entire hurricane area. It is natural to assume that at $r_0 = 0.5 r_1 \equiv r_{0.5}$ this is already the case. For this value of $r_{0.5}$ powers $W_{0.5}$ and $W_{P0.5}$ are equal to:

$$W_{0.5} = A_{0.5} v_{0.5}, \quad W_{P0.5} = A_{0.5} \frac{H_{P0}}{h_v} v_{0.5}. \tag{15}$$

According to graph in Fig. 2a of work [3] we have $u_{0.5} = 2u_1 = 2u_0$, $v_{0.5} = 2v_1$. In Eq. (15), it is assumed that precipitation path length H_{P0} changes little compared to $u_{0.5}$ and $v_{0.5}$. Using Eq. (12), we obtain

$$W_{0.5} = A_0 4 v_1 = 580 \text{ W m}^{-2}, \tag{16}$$

$$W_{P0.5} = A_0 \frac{H_{P0}}{h_v} 4 v_1 = 168 \text{ W m}^{-2}. \tag{17}$$

Subtracting Eq. (16) from Eq. (13) and Eq. (17) from Eq. (14), we have:

$$W_0 - W_{0.5} = A_0 (v_0 - 4v_1), \tag{18}$$

$$W_{P0} - W_{P0.5} = A_0 \frac{H_{P0}}{h_v} (v_0 - 4v_1). \tag{19}$$

From $v_1 = 7.5$ m s^{-1}, we obtain $4v_1 = 30$ m s^{-1}. This velocity, as follows from Eqs. (18) and (19), corresponds to the point, where the hurricane ceases to exist. This agrees well with observations (according to the operational definition of hurricane, it must have a sustained velocity exceeding 34 m s^{-1}). Within the error bars the constant term in (19), which is equal to 168 W m^{-2}, agrees well with the free term in Fig. 1d of work [2], where it is equal to 155 W m^{-2} (see Fig. 1).

Expressions (18) and (19) take into account that the power in the windwall does not determine total hurricane power. The power in the windwall does not exceed the entire hurricane power by more than twofold.

5. Conclusions

We derived the relationship between the gravitational power of precipitation and air velocity in the windwall from the previously developed theory of condensation-induced dynamics [3,5].

We emphasize that the gravitational power of precipitation exists irrespective of the dissipation of the kinetic energy of hurricanes (distinct from the interpretation given in work [2]). The hurricane power budget would remain the same even if precipitation occurred in free fall with rain drops not interacting with atmospheric air. In the real atmosphere, precipitation flux is $N_l V_l$, where N_l is molar density of liquid water (per unit air volume) and V_l is the mean droplet velocity. This flux is equal to the flux of ascending water vapor, $N_l V_l \sim N_v w$. Thus we have $N_l/N_v = w/V_l \sim 10^{-2}$–$10^{-3}$, which means that dissipation of kinetic energy of hurricane air owing to its interaction with falling droplets is several orders of magnitude smaller than the gravitational precipitation power W_P [5,6].

All energy of hurricane is concentrated in the windwall and the eye. This energy derives from condensation of water vapor accumulated within the hurricane area; in accordance with Bernoulli equation this energy increases logarithmically with narrowing radial air flow as it approaches the windwall. Velocity of air rotation in the windwall is the maximum velocity observed in the hurricane. Hurricane eye is the dominant source of turbulent friction, which significantly diminishes the windwall energy during its formation. The stationary cyclostrophic balance within the eye does not require a power input, which means that the eye and its kinetic energy can exist for a longer while independent of the windwall. Meanwhile disintegration of the windwall by the centrifugal force can only be prevented by the power of the radial air inflow, that is maintained by water vapor condensation. Unlike energy, the kinetic power of the hurricane is not concentrated within the windwall, but is distributed over the total hurricane area.

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References

[1] E.N. Rappaport, J.-G. Jiing, C.W. Landsea, S.T. Murillo, J.L. Franklin, Bull. Am. Meteorol. Soc. 93 (2012) 371.
[2] T. Sabuwala, G. Gioia, P. Chakraborty, Geophys. Res. Lett. 42 (2015) 3024.
[3] A.M. Makarieva, V.G. Gorshkov, Phys. Lett. A 375 (2011) 1053.
[4] V.G. Gorshkov, A.M. Makarieva, A.V. Nefiodov, J. Exp. Theor. Phys. 115 (2012) 723.
[5] A.M. Makarieva, V.G. Gorshkov, A.V. Nefiodov, Phys. Lett. A 378 (2014) 294.
[6] A.M. Makarieva, V.G. Gorshkov, A.V. Nefiodov, D. Sheil, A.D. Nobre, P. Bunyard, B.-L. Li, J. Atmos. Sci. 70 (2013) 2916.
[7] A.M. Makarieva, V.G. Gorshkov, A.V. Nefiodov, Phys. Lett. A 375 (2011) 2259.
[8] V.G. Gorshkov, Physical and Biological Bases of Life Stability: Man, Biota, Environment, Springer, 1995.