Hurricane's Maximum Potential Intensity and the Gravitational Power of Precipitation

A. M. Makarieva[1,2]*, V. G. Gorshkov[1,2], A.V. Nefiodov[1], A.V. Chikunov[3,4],
D. Sheil[5], A. D. Nobre[6], B.-L. Li[2]

[1]Theoretical Physics Division, Petersburg Nuclear Physics Institute, 188300 Gatchina, St. Petersburg, Russia [2]USDA-China MOST Joint Research Center for AgroEcology and Sustainability, University of California, Riverside 92521-0124, USA [3]Institute of World Ideas, Udaltsova street 1A, Moscow 119415, Russia [4]Princeton Institute of Life Sciences, Princeton, USA [5]Faculty of Environmental Sciences and Natural Resource Management, Norwegian University of Life Sciences, Ås, Norway [6]Centro de Ciência do Sistema Terrestre INPE, São José dos Campos, São Paulo 12227-010, Brazil

Abstract

The concept of Maximum Potential Intensity (MPI) is widely used in tropical cyclone research to estimate the minimum central pressure and the maximum velocity of tropical storms from environmental parameters. The MPI pressure derives from consideration of an idealized thermodynamic cycle, while the MPI velocity additionally requires information about real-time power and heat flows within the storm. Recently MPI velocity was proposed to be a substantial overestimate (by 10-30 percent) presumably neglecting the power needed to lift precipitating water (the gravitational power of precipitation). This conclusion did not involve a theoretical analysis of the MPI concept but was based on observed hurricane rainfall to estimate gravitational power of precipitation. However, since the MPI pressure estimate does explicitly account for lifting water, and the MPI velocity derives from this pressure, the question arises whether a correction to MPI velocity is required. Here we represent and examine the MPI derivations in their most general form and show that although a correction to MPI velocity is justified, it is an order of magnitude or so smaller than originally proposed. We show that the neglect of gravitational power of precipitation in the MPI velocity estimate was caused by the incomplete formulation of the general relationship between pressure work and dissipation of kinetic energy, taken per unit time and integrated over the storm. We highlight the importance of an internally consistent framework to estimate both storm energy and power and provide some perspectives for further investigating the role of moisture.

1 Introduction

Predicting hurricane intensity is a challenge in atmospheric research. Historically, much theoretical attention focused on finding upper limits on hurricane intensity (Malkus and Riehl, 1960; Holland, 1997; Camp and Montgomery, 2001). While *intensity* conventionally denotes maximum sustained velocity within a storm, early theoretical studies rather sought to estimate minimum central pressure known to be well correlated with maximum velocity.

Given that the hurricane is warmer than the ambient environment the idea was to retrieve the surface pressure deficit from this extra warmth assuming the existence of an unperturbed atmospheric top where pressures in the hurricane and the environment coincide. Since air pressure drops with altitude more slowly when the atmosphere is warm than when it is cold, to arrive at equal pressures at the top of the troposphere one must start from a lower surface

---

*Corresponding author. E-mail: ammakarieva@gmail.com

pressure in the warmer column. The height of the unperturbed top and the extra warmth of the hurricane compared to its environment uniquely determined the surface pressure deficit in the storm.

Emanuel (1986) advanced beyond this static approach by noting that pressure work (which produces the kinetic energy of wind) is constrained not only by the first law of thermodynamics, but also by the Bernoulli equation that derives from the equations of motion and continuity. Combining these two perspectives and additionally assuming that both the generation of kinetic energy and its dissipation (proportional to the cube of velocity) occur within the boundary layer, Emanuel (1986) thus linked *work* to *power* to estimate maximum velocity for a given central pressure. With its explicit formulae for calculating maximum hurricane velocity from environmental parameters, this approach became widely used in the tropical storm community.

Emanuel (1988, 1991, 1995, 1997) further advanced the MPI concept through several major modifications aimed to remedy the limitations of the original formulation. It was not before late 2000s that several evaluations of the concept were provided by independent theorists (Smith et al., 2008; Makarieva et al., 2010; Smith et al., 2014; Kieu, 2015). Those studies did not consider the MPI concept in its integrity, each focusing either on its dynamic or thermodynamic aspects. Recently, Sabuwala et al. (2015) proposed that the MPI velocity is a substantial overestimate since it presumably does not account for lifting water vapor.

Here it is essential to note that the MPI concept comprises two distinct approaches to storm's energetics (Fig. 1). One considers *work* in a steady-state thermodynamic cycle and kinetic energy variation along a closed streamline corresponding to that cycle (units J kg^{-1}) (Fig. 2). Another considers real-time heat flows and wind *power* (work per unit time) within the storm (units J s^{-1}). Emanuel (1986) calculated the minimum central pressure in the storm using the *work* approach; then used the obtained value in the *power* approach to calculate storm's maximum velocity. Power cannot be estimated from work without adding the scale of time. Furthermore, proceeding from an idealized Carnot cycle to real-time heat engines requires additional parameters describing the steady-state disequilibrium between the heat source and the working body (e.g., Curzon and Ahlborn, 1975). At the same time the work and power approaches are interdependent and should be combined coherently.

Here we analyze the MPI concept and show that while the energy needed for lifting water has been accounted for in the *work* approach, it is not incorporated in the *power* approach (Fig. 1). We show that in the presence of phase transitions the relationship between pressure work and kinetic energy dissipation is not the same in the work and power approaches. The omission of the gravitational power of precipitation in the MPI velocity estimate results from neglecting this difference.

2   Storm's budgets of energy and power

The MPI concept views the hurricane as a thermodynamic cycle consuming heat from the ocean (Fig. 2). Work performed in the cycle per unit mass of dry air is

$$- \oint \alpha_d dp = \varepsilon Q, \tag{1}$$

where $\alpha_d \equiv 1/\rho_d$ is the specific volume of dry air, ρ_d is dry air density, Q (J kg^{-1}) is heat input from the ocean per unit dry air mass and ε is the cycle's efficiency of converting heat to work.

Additionally, the MPI concept employs the Bernoulli equation

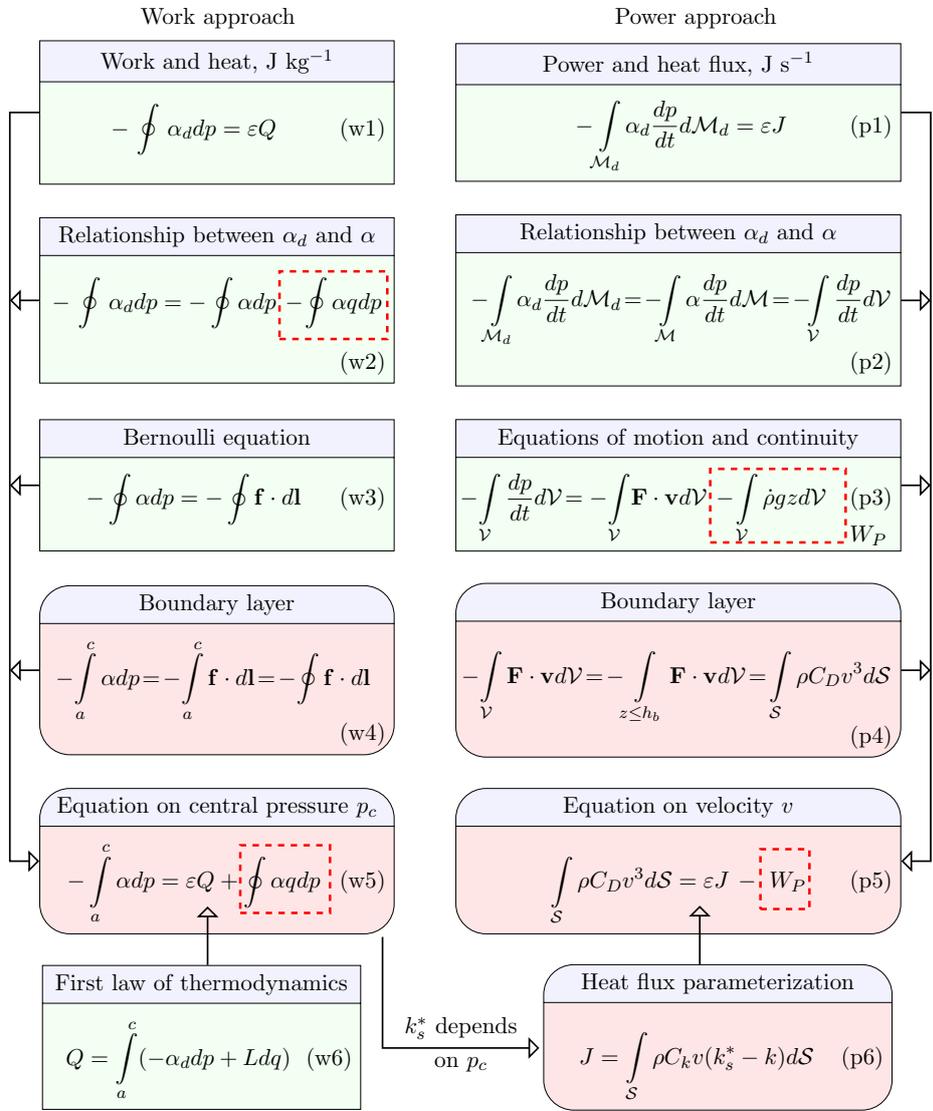$$d\left(\frac{v^2}{2}\right) + \alpha dp + g dz - \mathbf{f} \cdot d\mathbf{l} = 0, \tag{2}$$

Figure 1: Logical structure of the MPI concept. Rectangular boxes enclose generally valid statements, rounded boxes enclose assumptions specific to the MPI concept, $q \equiv \rho_v/\rho_d$, see text for the other notations. Dashed frames enclose terms accounting for the energy (left column) and power (right column) needed to lift water, W_P is the gravitational power of precipitation. Relationships in the left column are those given by Emanuel (1988), since in the original paper of Emanuel (1986) the difference between α and α_d and, hence, the energy needed to lift water was neglected. Term W_P in Eqs. (p3) and (p5) was never accounted for in the MPI derivations (Emanuel, 1986, 1988, 1991, 1995, 1997; Bister and Emanuel, 1998). Integrals in the left column refer to the thermodynamic cycle shown in Fig. 2.

where v is air velocity, $\alpha \equiv 1/\rho$, ρ is the density of moist air, $\mathbf{f}$ is the friction force per unit air mass and $d\mathbf{l} = \mathbf{v}dt$, see Emanuel (1986, Eq. 64) and Emanuel (1988, Eq. C1). From Eq. (2) we have

$$- \oint \alpha dp = - \oint \mathbf{f} \cdot d\mathbf{l}, \tag{3}$$

which can be interpreted as a "balance between pressure work and dissipation in steady flow" (Emanuel, 1988).

To proceed from work to real-time power, one has to replace dp in Eqs. (1) and (3) by dp/dt, where d/dt is material derivative, and integrate them over the entire hurricane occupying volume $\mathcal{V}$ with total air mass $\mathcal{M}$ and total dry air mass $\mathcal{M}_d$. Taking into account that $d\mathcal{V} = d\mathcal{M}_d/\rho_d = d\mathcal{M}/\rho$ we find using Eq. (1) (cf. Eqs. (w1) and (p1) in Fig. 1)

$$- \int_{\mathcal{M}_d} \alpha_d \frac{dp}{dt} d\mathcal{M}_d = - \int_{\mathcal{M}} \alpha \frac{dp}{dt} d\mathcal{M} = - \int_{\mathcal{V}} \frac{dp}{dt} d\mathcal{V} = \varepsilon J, \quad J \equiv \int_{z \leq h_b} \dot{Q} d\mathcal{M}_d. \tag{4}$$
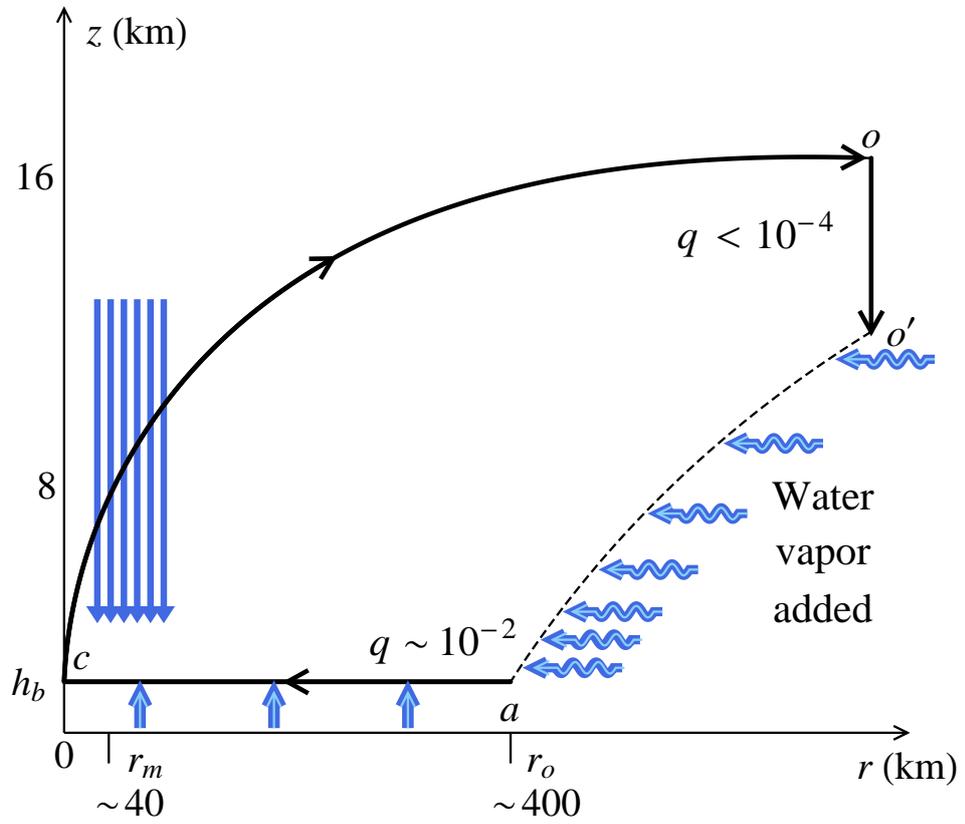
Figure 2: A hurricane in the MPI concept. Solid curve $a - c - o - o'$ is the actual streamline of air entering the hurricane at point a and leaving it at point o'; dashed curve $o' - a$ is the hypothetical path closing the thermodynamic cycle $a - c - o - o' - a$ with two isotherms, $a - c$ and $o - o'$, and two adiabats, $c - o$ and $o' - a$. As the air rises from c to o, water vapor condenses and precipitates, water vapor mixing ratio $q \equiv \rho_v/\rho_d$, where ρ_v is water vapor density, ρ_d is dry air density, declines by over two orders of magnitude. A major part of this lost water re-appears in the cycle (as shown by waved arrows) along the hypothetical $o' - a$ adiabat; in the real hurricane this imported moisture derives from evaporation outside the storm and is picked up as the storm moves through the atmosphere. The remaining part of moisture lost as rainfall is provided by evaporation from the sea surface (straight upward arrows). Straight downward arrows indicate the rainfall maximum that occurs in the vicinity of the radius of maximum wind $r = r_m$; r_o corresponding to point a is an external radius of the storm estimated to be approximately an order of magnitude larger than r_m (Emanuel, 1995, Table 1); $z = h_b$ is the height of the boundary layer.

Here J (W) is the total heat flow from the ocean into the hurricane, $\dot{Q}$ (W kg^{-1}) is the local heat source per unit mass of dry air, h_b is the height of the boundary layer beneath which this heat intake from the oceanic surface is assumed to occur (Fig. 2).

On the other hand, integrating the Bernoulli equation over $\mathcal{M}$ using the definition of material derivative

$$\frac{dp}{dt} = \mathbf{v} \cdot \nabla p, \tag{5}$$

and the continuity equation for gaseous air

$$\nabla \cdot (\rho \mathbf{v}) = \dot{\rho}, \tag{6}$$

where $\dot{\rho}$ (kg s^{-1} m^{-3}) is the volume-specific rate of phase transitions, we find

$$-\int_{\mathcal{V}} \frac{dp}{dt} d\mathcal{V} = -\int_{\mathcal{V}} \mathbf{F} \cdot \mathbf{v} d\mathcal{V} - \int_{\mathcal{V}} \dot{\rho} g z d\mathcal{V}, \tag{7}$$

see Appendix for details. Here $\mathbf{v}$ is the velocity vector for gaseous air, $\mathbf{F} \equiv \rho\mathbf{f}$ is friction force per unit air volume.

The second term in the right-hand part of Eq. (7) has the meaning of the gravitational power of precipitation. Since condensation predominantly occurs for $z > 0$ (and evaporation is concentrated at the surface $z = 0$), we have on average $\dot{\rho} < 0$ for $z > 0$. With gz being potential energy per unit mass, this term is positive and represents the rate at which condensation creates condensate with potential energy gz. For the real-time energy flows in the atmosphere pressure work per unit time given by the first term in Eq. (7) is *not* balanced by production/dissipation rate of kinetic energy alone but by the production/dissipation rate of kinetic energy *and* the gravitational power of precipitation.

To our knowledge, equation (7) and its derivation (Appendix) have not been previously described. While Pauluis et al. (2000) proposed that *total atmospheric power* should be equal to the sum of kinetic energy production/dissipation and the gravitational power of precipitation, they did not show that *total atmospheric power* is equal to $-\int_{\mathcal{V}}(dp/dt)d\mathcal{V}$. Thus, the idea that pressure work is balanced by kinetic energy dissipation as per Eq. (3) and the common use of continuity equations with a zero source term $\dot{\rho} = 0$ caused a misinterpretation of $-\int_{\mathcal{V}}(dp/dt)d\mathcal{V}$ for the *rate of kinetic energy production and dissipation* (e.g., Pauluis, 2015). This caused overestimation of the rate of kinetic energy production in the MPI concept as discussed below.

3   MPI velocity estimate

Combining Eqs. (4) and (7) we find

$$W_K = \left(\varepsilon - \frac{W_P}{J} \right) J, \tag{8}$$

where W_K (W) is the rate of kinetic energy production and dissipation and W_P (W) is the gravitational power of precipitation:

$$W_K \equiv -\int_{\mathcal{V}} \mathbf{F} \cdot \mathbf{v} d\mathcal{V}, \;\; W_P \equiv -\int_{\mathcal{V}} \dot{\rho} gz d\mathcal{V} > 0. \tag{9}$$

Equations (1)-(9) that we have so far considered are generally valid. Specific to the MPI concept is the assumption that dissipation of kinetic energy predominantly occurs in the boundary layer, such that

$$\oint \mathbf{f} \cdot d\mathbf{l} = \int_a^c \mathbf{f} \cdot d\mathbf{l} \tag{10}$$

in energy units (see, e.g., Eq. C4 of Emanuel (1988) where the last minor term is commonly neglected) and

$$W_K \equiv -\int_{\mathcal{M}} \mathbf{f} \cdot \frac{d\mathbf{l}}{dt} d\mathcal{M} = -\int_{z \leq h_b} \mathbf{F} \cdot \mathbf{v} d\mathcal{V} = \int_{\mathcal{S}} \rho C_D v^3 d\mathcal{S} \tag{11}$$

in power units (e.g., Eq. 7 of Emanuel, 1997). Expression under the last integral in Eq. (11) is the local dissipation rate in the boundary layer per unit surface area, $C_D \sim 10^{-3}$ is a dimensionless coefficient, the integration is made over area $\mathcal{S}$ occupied by the hurricane.

For the heat input from the ocean the MPI concept uses the following relationship (see, e.g., Emanuel, 1995, Eq. 3):

$$J = \int_{\mathcal{S}} \rho C_k v(k_s^* - k) d\mathcal{S}, \tag{12}$$

where $C_k \sim 10^{-3}$, k_s^* (J kg^{-1}) is saturated enthalpy of air at surface temperature and k is the actual enthalpy of air in the boundary layer.

Combining Eqs. (8), (11) and (12), we find

$$\int\limits_{\mathcal{S}} \rho C_D v^3 d\mathcal{S} = \varepsilon_K \int\limits_{\mathcal{S}} \rho C_k v(k_s^* - k)d\mathcal{S}, \quad \varepsilon_K \equiv \varepsilon - \frac{W_P}{J} < \varepsilon. \tag{13}$$

A major assumption within the MPI concept is that Eq. (13) (but with ε_K replaced by ε) holds for the expressions under the integral in the region of maximum velocities (Emanuel, 1997), such that from Eq. (13) we would have

$$v_{\max}^2 = \varepsilon_K \frac{C_k}{C_D}(k_s^* - k). \tag{14}$$

We discuss this assumption below. Here we note that ε and ε_K are not local characteristics but those of the thermodynamic *cycle* and the considered closed air trajectory *as a whole*. Therefore, if the major inputs into Eq. (13) are indeed made in the region of $v = v_{\max}$, as assumed in the MPI concept, the reduction of efficiency and replacement of ε by ε_K in Eq. (13) will also apply to the velocity estimate $v_{\max}$ (14).

We can represent the gravitational power of precipitation as

$$W_P = PgH_P, \tag{15}$$

where H_P is the mean height at which condensation occurs (Gorshkov and Dol'nik, 1980; Pauluis, 2011; Makarieva et al., 2013) and P (kg s^{-1}) is total precipitation over area $\mathcal{S}$ of the storm.

For the oceanic heat input we have

$$J = J_S + J_L = (B + 1)PL, \tag{16}$$

where J_S and J_L are fluxes of sensible and latent heat, respectively; $B \equiv J_S/J_L$ is the Bowen ratio, $L = 2.5 \times 10^6$ J kg^{-1} is the latent heat of vaporization and P is rainfall here assumed to be equal to the flux of evaporation to be consistent with the steady-state MPI concept.

From Eqs. (13), (15) and (16) we find

$$\varepsilon_K = \varepsilon - \frac{gH_P}{(B + 1)L}. \tag{17}$$

Mean precipitation height H_P can be calculated from the equation of moist adiabat and depends on surface temperature T_s, the incompleteness of condensation ζ and, to a lesser degree, on surface relative humidity (Makarieva et al., 2013). We assume that moist air having temperature T_s and relative humidity 80% at the surface first rises dry adiabatically up to height z_1 where water vapor becomes saturated. Then it rises moist adiabatically to z_2, where condensation ceases. At z_2 the air preserves share ζ of its initial water vapor content, $\zeta \equiv \gamma(z_2)/\gamma_s = \gamma(z_2)/\gamma(z_1)$. Here $\gamma \equiv p_v/p$, where p_v is water vapor partial pressure and p is air pressure. Moist adiabatic distributions of $\gamma(z)$ and $p(z)$ with $p_s = 1000$ hPa, where subscript s denotes surface values, were calculated according to Eqs. (A3)-(A5) of Makarieva et al. (2013)

For T_s ranging from 260 to 310 K and for ζ ranging from 0.001 (almost complete condensation) to 1/2, we estimated mean condensation height as

$$H_P(T_s, \zeta) = \frac{1}{\gamma(z_2) - \gamma(z_1)} \int_{z_1}^{z_2} z\frac{\partial \gamma}{\partial z}dz. \tag{18}$$

Height H_P grows with increasing temperature but even for complete removal of water vapor from the air, which always occurs if the air rises above 16 km, as is often the case in hurricanes, H_P does not exceed 6 km (Fig. 3).

For a typical Bowen ratio of 1/3 (Jaimes et al., 2015) with $H_P = 6$ km we obtain from Eq. (17) that the gravitational power of precipitation reduces ε_K compared to ε by $gH_P/L \sim 0.018$ at most. With ε for hurricanes viewed as Carnot cycles being around 0.3 (Emanuel, 1986; DeMaria and Kaplan, 1994), this represents a 6% reduction to ε and a 3% reduction to velocity $v_{\max}$ (14). For circulations with a smaller ε the relative reduction would be larger.
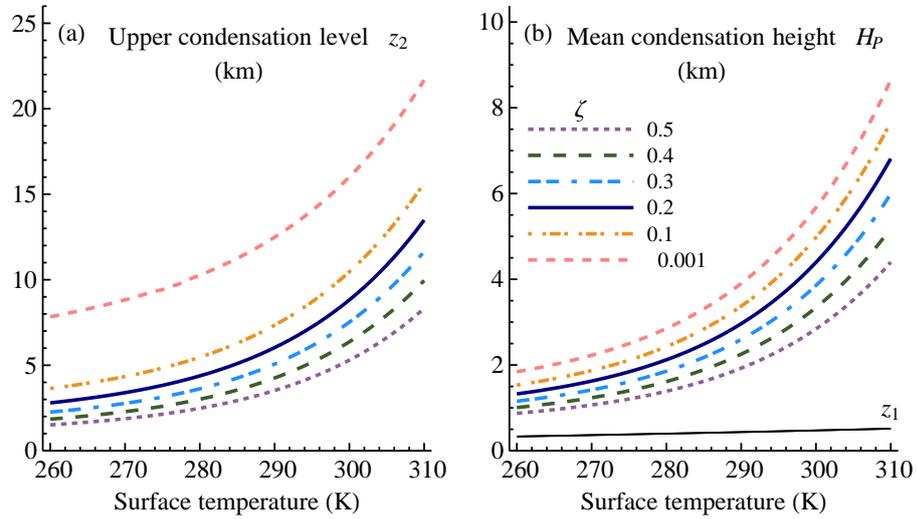
Figure 3: The upper condensation level z_2 (a), lower condensation level z_1 and mean condensation height H_P (18) (b), as dependent on surface temperature T_s and incompleteness of condensation ζ calculated following Makarieva et al. (2013).

4   Results of Sabuwala et al. (2015)

Sabuwala et al. (2015) investigated how accounting for the gravitational power of precipitation can reduce maximum velocity compared to its MPI estimate. They did not present a theoretical estimate of ε_K but used instead empirical TRMM data on hurricane rainfall in the vicinity of maximum velocity. Their conclusion was a 10-30% reduction in velocity, which is significantly higher than our estimate of 3%.

This discrepancy appears linked to two problems in the analysis of Sabuwala et al. (2015). First, they used local values of rainfall measured in the vicinity of maximum velocity, while the reduction pertains to the considered cycle as a whole and thus should be estimated using mean rainfall within the storm. Mean rainfall within the outermost closed isobar as estimated from TRMM data is several times lower than maximum rainfall in the vicinity of the radius of maximum winds. For North Atlantic hurricanes it is 2 mm hr^{-1} within 400 km and about 8 mm hr^{-1} within 100 km which includes the radius of maximum wind (see, e.g., Fig 4j of Makarieva et al., 2017a).

Second, of this rainfall only about one quarter or third is represented by moisture evaporated within the outermost closed isobar. The major part of precipitating water is imported from outside (Makarieva et al., 2017a). The MPI concept does not explicitly account for this imported moisture as it views the hurricane as a steady-state thermodynamic cycle with moisture provided locally by evaporation from the ocean. This imported moisture is, however, implicitly accounted for by considering the hypothetical adiabat $o' - a$ to be part of the hurricane's thermodynamic cycle (Fig. 2).

Along $o' - a$ the moisture content of the hypothetically descending air rises by over two orders of magnitude. In this region, i.e. outside the radius of the outermost closed isobar r_o (Fig. 2), there is a characteristic *clear-sky moat* (Frank, 1977; Makarieva et al., 2017a). Thus this increase of moisture content cannot occur due to "mixing with cloudy air" (cf. Pauluis and Zhang, 2017). Indeed, most moisture condensing within the hurricane precipitates and cannot serve as a source of water vapor for the descending air. The vertical distribution of humidity along the $o' - a$ path is in fact provided by evaporation and convection outside the storm. These moisture stores are picked up by the hurricane as it moves through the atmosphere. As the moisture is imported with its own gravitational energy, the storm does not need to spend energy and power to raise this water.

This issue was addressed by Emanuel (1988) within the work approach (see the left column in Fig. 1). Comparing Eqs. (3) and (7) (cf. also Eqs. (w3) and (p3) in Fig. 1) we can

see that while the integrals of αdp and $\mathbf{f} \cdot d\mathbf{l}$ (J kg^{-1}) over closed streamlines coincide, the integrals of $\alpha dp/dt$ and $\mathbf{f} \cdot d\mathbf{l}/dt$ (W kg^{-1}) over the hurricane mass are not equal. The reason is that integration over the closed streamline is made for a constant unit mass. Meanwhile in the real-time atmosphere in the presence of phase transitions there is simultaneously more gas rising and expanding (positive work) than descending and compressing (negative work). The difference between these amounts of gas is what accounts for the gravitational power of precipitation.

Accordingly, Emanuel (1988) showed that work of the thermodynamic cycle is equal to the sum of kinetic energy generation and the net work of rising (expanding) versus compressing (descending) water vapor expressed by the term $\oint \alpha q dp$, see Eqs. (w2) and (w5) in Fig. 1. Emanuel (1988) indicated that the term accounting for water lifting energy is proportional to the difference in the water profiles of the air rising along $c - o$ path and the environmental air along the hypothetical path $o' - a$ (Emanuel, 1988, see Appendix C and Eq. (C12)). Recently Pauluis and Zhang (2017) repeated these derivations to retrieve the energy needed to lift water from a numerical hurricane model. Neither Emanuel (1988) nor Pauluis and Zhang (2017) estimated the contribution of W_P theoretically (this requires an estimate of H_P as per Fig. 3).

Thus, within the work approach which allows the estimation of hurricane's pressure profile and its minimum central pressure p_c (which in turn impacts the value of saturated enthalpy k_s^* in the expression for $v_{\max}$, see Eq. (w5) in Fig. 1) the energy needed to lift water was accounted for by Emanuel (1988). This requires recognizing the difference between α and α_d (Fig. 1) which was not accounted for in the original paper by Emanuel (1986) and in subsequent papers (e.g., Emanuel, 1991). Indeed, while local values of α_d and α are very close (the difference is of the order of water vapor mixing ratio $q \sim 10^{-2}$), their integrals over the closed cycle differ by an amount comparable to the two integrals themselves: all terms in Eq. (w2) in Fig. 1 can be of the same order of magnitude. In the power approach the gravitational power of precipitation has not been so far accounted for.

Returning to the estimates of Sabuwala et al. (2015), overestimating the surface-specific rainfall in (15) is equivalent to overestimating H_P in (17) by the same factor. Using local rainfall instead of mean hurricane rainfall (the overestimate factor of about 4) and all rainfall instead of rainfall provided by evaporation from the hurricane (the overestimate factor of 3-4), they should have overestimated the actual correction to $v_{\max}$ by an order of magnitude. This explains the difference between their maximum proposed reduction of 30% and our estimate of 3%.

5   Discussion

We have shown that the reduction in MPI velocity estimate associated with the gravitational power of precipitation was previously overestimated but that this can be addressed using Eq. (7). This equation shows that, unlike in the work approach where the work due to pressure is balanced by frictional dissipation (Eq. (w3) in Fig. 1), in the power approach work per unit time of the pressure gradient forces is balanced by the rate of frictional dissipation of kinetic energy plus the gravitational power of precipitation (Eq. (p3) in Fig. 1).

The relationships summarized in Fig. (1), and the results we derived from them, including Eq. (7), are general as they are based on only the first law of thermodynamics and the equations of motion and continuity. This approach thus avoids additional assumptions, for example, the assumptions of thermal wind and of hydrostatic balance as employed by Emanuel (1986) following the ideas of Kleinschmidt Jr. (1951). Our relationships can therefore be used to check the consistency of other derivations within this general framework. Given the prominence of the MPI concept in the tropical storms research we believe that it would be useful to re-visit the other assumptions within the concept in the view of the constraints imposed by a simultaneous consideration of Eqs. (w1)-(p6) in Fig. 1. Below we

outline several perspectives for such an analysis.

First, the MPI concept is based on a relationship between angular momentum and moist entropy used to justify the transition from the integral equation (11) to the local equation (14). This logic is based on the following assumptions: 1) that air at $z = h_b$ (height of the boundary layer) is saturated; 2) that it is isothermal and 3) that air at $z = 0$ is also isothermal (for details see Emanuel, 1986, in particular, the first two unnumbered equations on page 589, right column). At the same time, Eq. (w6) in Fig. 1 shows that the air moving from a to c increases its water vapor content, i.e. q grows from a to c. It is easy to see that these four conditions, air saturated and isothermal at $z = h_b$, air isothermal at $z = 0$ and q at $z = h_b$ growing from a to c are not compatible with each other. (In brief, for the isothermy at both $z = 0$ and $z = h_b$ to be satisfied, the lapse rate below $z = h_b$ must be the same everywhere; i.e. it must be dry adiabatic (since at different q the moist adiabatic lapse rate will not be the same). However, saturation height z_1 for a given temperature depends on q: the higher the q, the lower the saturation height; when q is saturated, $z_1 = 0$. So, if q increases from a to c, the level at which it becomes saturated diminishes. Thus, if at point c the air is saturated at $z = h_b$ but not below, it cannot be saturated at point a (with a smaller q) at the same height $z = h_b$, thus the first condition is violated.)

Second, since the MPI concept assumes that the surface air is isothermal, and there is no other reason for this isothermy rather than the isothermal oceanic surface, the concept presumes that the temperature of the surface air and the ocean coincide (see, e.g., Holland, 1997, Table 1, third row, forth column). This means that the flux of sensible heat J_S from the ocean to the hurricane is absent. All energy input consists in the flux of latent heat J_L. However, the first law of thermodynamics (w6) in Fig. 1 shows that total heat Q input into the air as it moves from a to c is not confined to latent heat alone $Q_L = \int_a^c L dq$. It also includes the term $\alpha_d dp$: the air expands but remains isothermal gaining heat from somewhere beyond latent heat input. Thus, while the MPI assumption that the temperature of surface air coincides with that of the ocean prescribes that sensible heat input is zero, latent heat alone cannot account for the observed isothermal expansion of air parcels. To our knowledge, this problem was not explicitly acknowledged, but Bister and Emanuel (1998) suggested that there is an extra source of heat: it is the dissipation of kinetic energy generated within the hurricane (see discussion by Makarieva et al., 2010; Bister et al., 2011; Kieu, 2015).

Bister and Emanuel (1998) formulated this extra heat source within the *power* approach. Considering it within the *work* approach reveals the following issue. Generation of kinetic energy, αdp and sensible heat input $\alpha_d dp$ practically concide at the isotherm $a - c$ because of the smallness of q. If at some point x we have $-\int_a^x \alpha_d dp = -\int_a^x \alpha dp = -\int_a^x \mathbf{f} \cdot d\mathbf{l}$, this means, according to the Bernoulli equation (2), that $\int_a^x dv^2 = 0$. In other words, if the energy generated by pressure work $-\alpha dp$ is dissipated by friction $-\mathbf{f} \cdot d\mathbf{l}$ to account for the missing sensible heat $\alpha_d dp$, air velocity in the hurricane cannot rise as the air moves from a towards the center, which contradicts observations. This emphasizes the need to jointly consider both work and power approaches when formulating MPI (Makarieva et al., in preparation).

Third, we believe that the MPI concept may have a broader dynamic interpretation not confined to the Carnot cycle or a steady-state case it has so far been linked to. As the air leaves the boundary layer, it must have sufficient energy to flow away from the hurricane. This energy can be provided by a pressure gradient in the upper atmosphere (Makarieva et al., 2017c): if the air pressure in the column above the area of maximum wind is higher than in the ambient environment, it will accelerate the air outward. However, a significant pressure deficit at the surface precludes the formation of a significant pressure surplus aloft. On the other hand, namely this pressure deficit is what accelerates the hurricane air in the boundary layer. Given that the air as it leaves the boundary layer possesses a certain kinetic energy, we can require that this energy is spent to overcome the negative pressure gradient in the upper atmosphere. Thus, the air will be flowing outward not at the expense of a pressure gradient but against it, at the expense of accumulated kinetic energy (propelled by the centrifugal force). Maximum intensity in this case will be determined from the condition that the kinetic

energy accumulated at the expense of pressure deficit at the surface is enough to overcome the pressure surplus in the upper atmosphere for the air to flow away. This condition should be a valid upper limit to hurricane intensity for any type of air interaction with the ocean, steady and non-steady circulations alike. As we discuss elsewhere, the MPI concept in its dynamic part expresses these ideas and thus, in a properly modified form, should have a broader generality than so far assumed (Makarieva et al., in preparation).

Forth, the hurricane can be divided into two zones. One is $r \geq r_m$ where the air moves across the isobars and the kinetic power is generated. The other is the eye, which can be approximated by solid body rotation (e.g., Emanuel, 1997) with a non-zero energy store but zero power. The pressure drop observed across the hurricane likewise consists of two parts: the pressure drop within the eye represents a *store of potential energy* equal to the kinetic energy of the eye; no power is generated here. The pressure drop from the ambient environment to the outer border of maximum wind, which is less than a half of the total pressure drop, is what actually generates hurricane power (of which a minor part goes to maintain the slowly dissipating energy store within the eye). Recognition of this spatial division into the *energy* and *power* zones is essential for understanding hurricane's energetics.

Fifth, and arguably most important, Sabuwala et al. (2015) demonstrated using empirical data that hurricane intensity is correlated with rainfall intensity. Furthermore, there is a growing body of evidence revealing correlation between external moisture supply and hurricane power (Krishnamurti et al., 1993, 1998; Fritz and Wang, 2014; Ermakov et al., 2014, 2015; Fujiwara et al., 2017). However, a qualitative let alone quantitative explanation of these patterns remains elusive. As mentioned by Sabuwala et al. (2015), one explanation found in the literature is that a higher hurricane intensity is caused by a higher intensity of latent heat release. Besides storms research, this logic is employed in a number of studies exploiting the role of moisture in ocean-to-land monsoon-like circulations (Levermann et al., 2009; Herzschuh et al., 2014; Levermann et al., 2016; Boers et al., 2017). However, as recently pointed out by Boos and Storelvmo (2016b), who quoted Emanuel et al. (1994), see also Boos and Storelvmo (2016a), the idea that a more intense release of latent heat makes the atmosphere warmer represents *"an influential and lengthy dead-end road in atmospheric science"*. Indeed, the steady-state pressure gradients associated with latent heat release are independent of the intensity of rainfall; they only depend on the steady-state difference in the amounts of moisture between the rising air and its environment. Again, work/energy and power approaches are confused here.

Thus, a more intense release of latent heat cannot explain the observed correlation between the circulation intensity and rainfall in either hurricane or monsoon studies. At this moment the only concept that provides a quantitative explanation for this pattern is condensation-induced dynamics (Makarieva et al., 2014, 2015). Here the key process is the positive feedback between the radial air motion and the pressure drop at the surface associated with condensation and hydrostatic adjustment. As the air streams towards the hurricane center and ascends, the water vapor condenses and the air pressure drops. The key numerical scale for this process is the saturated partial pressure of water vapor at the surface (40 hPa at 30 °C), which gives the maximum pressure drop in the *power* region of the hurricane. This approach requires greater attention from the meteorological community.

Appendix A: Deriving Equation (7)

For any quantity X we can write

$$\rho(\mathbf{v} \cdot \nabla)X = (\nabla \cdot \mathbf{v})X\rho - X\nabla \cdot (\rho\mathbf{v}) = (\nabla \cdot \mathbf{v})X\rho - X\dot{\rho}, \tag{A.1}$$

where in the last equality the steady-state continuity equation (6) has been applied. Using the definition of material derivative (5) and the divergence theorem we obtain from (A.1)

$$\int_{\mathcal{V}} \rho \frac{dX}{dt} d\mathcal{V} = -\int_{\mathcal{V}} \dot{\rho} X d\mathcal{V}. \tag{A.2}$$

We have assumed that the circulation is closed such that $\int \mathbf{v} \cdot \mathbf{n} dS = 0$, where S is the bounding surface of the circulation and $\mathbf{n}$ is the unit normal vector.

Bernoilli equation (2) in the form

$$\rho \frac{dK}{dt} = -\frac{dp}{dt} + \rho \mathbf{g} \cdot \mathbf{w} + \mathbf{F} \cdot \mathbf{v}, \tag{A.3}$$

where $K \equiv v^2/2$ is kinetic energy per unit air mass, is obtained from the steady-state equations of motion

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \rho \mathbf{g} + \mathbf{F}, \tag{A.4}$$

by taking their scalar product with gas velocity vector $\mathbf{v}$.

Noting that $\mathbf{g} = -g\nabla z$ and using (A.2) with $X = z$ we obtain for the volume integral of the second term in the right-hand part of (A.3)

$$\int_{\mathcal{V}} \rho \mathbf{g} \cdot \mathbf{w} d\mathcal{V} = -\int_{\mathcal{V}} \rho g \frac{dz}{dt} d\mathcal{V} = \int_{\mathcal{V}} \dot{\rho} g z d\mathcal{V}. \tag{A.5}$$

Using (A.2) with $X = K$ and integrating (A.3) over $\mathcal{V}$ we find

$$\underbrace{-\int_{\mathcal{V}} \frac{dp}{dt} d\mathcal{V}}_{A} = \underbrace{-\int_{\mathcal{V}} \mathbf{F} \cdot \mathbf{v} d\mathcal{V}}_{B} \underbrace{-\int_{\mathcal{V}} \dot{\rho} g z d\mathcal{V}}_{C} \underbrace{-\int_{\mathcal{V}} \dot{\rho} K d\mathcal{V}}_{D}. \tag{A.6}$$

Work of atmospheric pressure gradients per unit time (A) is balanced by dissipation of kinetic energy (B), production of the gravitational power of precipitation (C) and production of the kinetic power of gas that is converted to liquid (D). We have neglected the condensate loading in the equations of motion (A.4), which implies that all condensate is instantaneously removed from the atmosphere. Consideration of condensate loading requires a specification of how it interacts with the air (Makarieva et al., 2017b). The resulting term is however small relative to C and can be neglected.

Since typical values of z where condensation occurs are $z \sim 5$ km, we have $gz \gg K$ for any realistic values of v. Even in hurricanes with $v \sim 60$ m s^{-1}, K is only about 4% of gz. Thus term D in (A.6) can be neglected without losing accuracy.

Acknowledgment

References

Bister, M. and K. A. Emanuel, 1998: Dissipative heating and hurricane intensity. *Meteorol. Atmos. Phys.*, **65**, 233–240, doi:10.1007/BF01030791.

Bister, M., N. Renno, O. Pauluis, and K. Emanuel, 2011: Comment on Makarieva et al. 'A critique of some modern applications of the Carnot heat engine concept: the dissipative heat engine cannot exist'. *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, **467**, 1–6, doi:10.1098/rspa.2010.0087.

Boers, N., N. Marwan, H. M. J. Barbosa, and J. Kurths, 2017: A deforestation-induced tipping point for the South American monsoon system. *Scientific Reports*, **7 (41489)**, doi:doi:10.1038/srep41489.

Boos, W. R. and T. Storelvmo, 2016a: Near-linear response of mean monsoon strength to a broad range of radiative forcings. *Proc. Natl. Acad. Sci.*, **113 (6)**, 1510–1515, doi:10.1073/pnas.1517143113.

Boos, W. R. and T. Storelvmo, 2016b: Reply to Levermann et al.: Linear scaling for monsoons based on well-verified balance between adiabatic cooling and latent heat release. *Proc. Natl. Acad. Sci.*, **113 (17)**, E2350–E2351, doi:10.1073/pnas.1603626113.

Camp, J. P. and M. T. Montgomery, 2001: Hurricane maximum intensity: Past and present. *Mon. Weather Rev.*, **129**, 1704–1717, doi:10.1175/1520-0493(2001)129<1704:HMIPAP>2.0.CO;2.

Curzon, F. L. and B. Ahlborn, 1975: Efficiency of a Carnot engine at maximum power output. *Am. J. Phys.*, **43**, 22–24, doi:10.1119/1.10023.

DeMaria, M. and J. Kaplan, 1994: Sea surface temperature and the maximum intensity of Atlantic tropical cyclones. *J. Climate*, **7**, 1324–1334, doi:10.1175/1520-0442(1994)007<1324:SSTATM>2.0.CO;2.

Emanuel, K. A., 1986: An air-sea interaction theory for tropical cyclones. Part I: Steady-state maintenance. *J. Atmos. Sci.*, **43**, 585–604, doi:10.1175/1520-0469(1986)043<0585:AASITF>2.0.CO;2.

Emanuel, K. A., 1988: The maximum intensity of hurricanes. *J. Atmos. Sci*, **45**, 1143–1155, doi:10.1175/1520-0469(1988)045<1143:TMIOH>2.0.CO;2.

Emanuel, K. A., 1991: The theory of hurricanes. *Annu. Rev. Fluid Mech.*, **23**, 179–196, doi:10.1146/annurev.fl.23.010191.001143.

Emanuel, K. A., 1995: Sensitivity of tropical cyclones to surface exchange coefficients and a revised steady-state model incorporating eye dynamics. *J. Atmos. Sci*, **52**, 3969–3976, doi:10.1175/1520-0469(1995)052<3969:SOTCTS>2.0.CO;2.

Emanuel, K. A., 1997: Some aspects of hurricane inner-core dynamics and energetics. *J. Atmos. Sci*, **54**, 1014–1026, doi:10.1175/1520-0469(1997)054<1014:SAOHIC>2.0.CO;2.

Emanuel, K. A., J. David Neelin, and C. S. Bretherton, 1994: On large-scale circulations in convecting atmospheres. *Q. J. R. Meteorol. Soc.*, **120**, 1111–1143, doi:10.1002/qj.49712051902.

Ermakov, D. M., E. A. Sharkov, and A. P. Chernushich, 2014: The role of tropospheric advection of latent heat in the intensification of tropical cyclones. *Issledovanie Zemli iz Kosmosa, (Earth Research from Space)*, **4**, 3–15, doi:10.7868/S0205961414040046, (in Russian).

Ermakov, D. M., E. A. Sharkov, and A. P. Chernushich, 2015: Satellite radiothermovision of atmospheric mesoscale processes: case study of tropical cyclones. *Int. Arch. Photogramm. Remote Sens. Spatial Inf. Sci.*, **XL-7/W3**, 179–186, doi:10.5194/isprsarchives-XL-7-W3-179-2015.

Frank, W. M., 1977: The structure and energetics of the tropical cyclone I. Storm structure. *Mon. Weather Rev.*, **105**, 1119–1135, doi:10.1175/1520-0493(1977)105<1119:TSAEOT>2.0.CO;2.

Fritz, C. and Z. Wang, 2014: Water vapor budget in a developing tropical cyclone and its implication for tropical cyclone formation. *J. Atmos. Sci.*, **71**, 4321–4332, doi:10.1175/JAS-D-13-0378.1.

Fujiwara, K., R. Kawamura, H. Hirata, T. Kawano, M. Kato, and T. Shinoda, 2017: A positive feedback process between tropical cyclone intensity and the moisture conveyor belt assessed with lagrangian diagnostics. *J. Geophys. Res. Atmos.*, **122**, 12 502–12 521, doi:10.1002/2017JD027557.

Gorshkov, V. G. and V. R. Dol'nik, 1980: Energetics of the biosphere. *Soviet Physics Uspekhi*, **23**, 386–408, doi:10.1070/PU1980v023n07ABEH005117.

Herzschuh, U., J. Borkowski, J. Schewe, S. Mischke, and F. Tian, 2014: Moisture-advection feedback supports strong early-to-mid Holocene monsoon climate on the eastern Tibetan Plateau as inferred from a pollen-based reconstruction. *Palaeogeography, Palaeoclimatology, Palaeoecology*, **402**, 44 – 54, doi:https://doi.org/10.1016/j.palaeo.2014.02.022.

Holland, G. J., 1997: The maximum potential intensity of tropical cyclones. *J. Atmos. Sci.*, **54**, 2519–2541, doi:10.1175/1520-0469(1997)054<2519:TMPIOT>2.0.CO;2.

Jaimes, B., L. K. Shay, and E. W. Uhlhorn, 2015: Enthalpy and momentum fluxes during Hurricane Earl relative to underlying ocean features. *Mon. Weather Rev.*, **143**, 111–131, doi:10.1175/MWR-D-13-00277.1.

Kieu, C., 2015: Revisiting dissipative heating in tropical cyclone maximum potential intensity. *Quart. J. Roy. Meteorol. Soc.*, **141**, 2497–2504, doi:10.1002/qj.2534.

Kleinschmidt Jr., E., 1951: Grundlagen einer Theorie der tropischen Zyklonen. *Archiv für Meteorologie, Geophysik und Bioklimatologie, Serie A*, **4**, 53–72, doi:10.1007/BF02246793.

Krishnamurti, T. N., H. S. Bedi, and K. Ingles, 1993: Physical initialization using SSM/I rain rates. *Tellus A*, **45**, 247–269, doi:10.1034/j.1600-0870.1993.t01-3-00001.x.

Krishnamurti, T. N., W. Han, and D. Oosterhof, 1998: Sensitivity of hurricane intensity forecasts to physical initialization. *Meteorol. Atmos. Phys.*, **65**, 171–181, doi:10.1007/BF01030786.

Levermann, A., V. Petoukhov, J. Schewe, and H. J. Schellnhuber, 2016: Abrupt monsoon transitions as seen in paleorecords can be explained by moisture-advection feedback. *Proc. Natl. Acad. Sci.*, **113 (17)**, E2348–E2349, doi:10.1073/pnas.1603130113.

Levermann, A., J. Schewe, V. Petoukhov, and H. Held, 2009: Basic mechanism for abrupt monsoon transitions. *Proc. Natl. Acad. Sci.*, **106 (49)**, 20 572–20 577, doi:10.1073/pnas.0901414106.

Makarieva, A. M., V. G. Gorshkov, B.-L. Li, and A. D. Nobre, 2010: A critique of some modern applications of the Carnot heat engine concept: the dissipative heat engine cannot exist. *Proc. R. Soc. A*, **466**, 1893–1902, doi:10.1098/rspa.2009.0581.

Makarieva, A. M., V. G. Gorshkov, and A. V. Nefiodov, 2014: Condensational power of air circulation in the presence of a horizontal temperature gradient. *Phys. Lett. A*, **378**, 294–298, doi:10.1016/j.physleta.2013.11.019.

Makarieva, A. M., V. G. Gorshkov, and A. V. Nefiodov, 2015: Empirical evidence for the condensational theory of hurricanes. *Phys. Lett. A*, **379**, 2396–2398, doi: 10.1016/j.physleta.2015.07.042.

Makarieva, A. M., V. G. Gorshkov, A. V. Nefiodov, A. V. Chikunov, D. Sheil, A. D. Nobre, and B.-L. Li, 2017a: Fuel for cyclones: The water vapor budget of a hurricane as dependent on its movement. *Atmospheric Research*, **193**, 216–230, doi: 10.1016/j.atmosres.2017.04.006.

Makarieva, A. M., V. G. Gorshkov, A. V. Nefiodov, D. Sheil, A. D. Nobre, P. Bunyard, and B.-L. Li, 2013: The key physical parameters governing frictional dissipation in a precipitating atmosphere. *J. Atmos. Sci.*, **70**, 2916–2929, doi:10.1175/JAS-D-12-0231.1.

Makarieva, A. M., V. G. Gorshkov, A. V. Nefiodov, D. Sheil, A. D. Nobre, P. Bunyard, P. Nobre, and B.-L. Li, 2017b: The equations of motion for moist atmospheric air. *J. Geophys. Res. Atmos.*, **122**, 7300–7307, doi:10.1002/2017JD026773.

Makarieva, A. M., V. G. Gorshkov, A. V. Nefiodov, D. Sheil, A. D. Nobre, P. L. Shearman, and B.-L. Li, 2017c: Kinetic energy generation in heat engines and heat pumps: The relationship between surface pressure, temperature and circulation cell size. *Tellus A*, **69**, 1272 752, doi:10.1080/16000870.2016.1272752.

Makarieva et al., in preparation: Hurricane's Maximum Potential Intensity, hypercanes and fluxes of sensible and latent heat.

Malkus, J. S. and H. Riehl, 1960: On the dynamics and energy transformations in steady-state hurricanes. *Tellus*, **12**, 1–20, doi:10.1111/j.2153-3490.1960.tb01279.x.

Pauluis, O., 2011: Water vapor and mechanical work: A comparison of Carnot and steam cycles. *J. Atmos. Sci.*, **68**, 91–102, doi:10.1175/2010JAS3530.1.

Pauluis, O., V. Balaji, and I. M. Held, 2000: Frictional dissipation in a precipitating atmosphere. *J. Atmos. Sci.*, **57**, 989–994, doi:10.1175/1520-0469(2000)057<0989:FDIAPA>2.0.CO;2.

Pauluis, O. M., 2015: The global engine that could. *Science*, **347**, 475–476, doi: 10.1126/science.aaa3681.

Pauluis, O. M. and F. Zhang, 2017: Reconstruction of thermodynamic cycles in a high-resolution simulation of a hurricane. *Journal of the Atmospheric Sciences*, **74**, 3367–3381, doi:10.1175/JAS-D-16-0353.1.

Sabuwala, T., G. Gioia, and P. Chakraborty, 2015: Effect of rainpower on hurricane intensity. *Geophys. Res. Lett.*, **42**, 3024–3029, doi:10.1002/2015GL063785.

Smith, R. K., M. T. Montgomery, and J. Persing, 2014: On steady-state tropical cyclones. *Quart. J. Roy. Meteorol. Soc.*, **140**, 2638–2649, doi:10.1002/qj.2329.

Smith, R. K., M. T. Montgomery, and S. Vogl, 2008: A critique of Emanuel's hurricane model and potential intensity theory. *Q. J. R. Meteorol. Soc.*, **134**, 551–561.