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GEOPHYSICS

The Greenhouse Effect and the Stability of the Global Mean Surface Temperature

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The Earth's surface temperature and climate depend on values of the greenhouse effect and the planetary albedo. The greenhouse effect f is defined as the difference between the intensities of the observed surficial thermal radiation q and the space-oriented radiation of the atmosphere's upper radiation layer q_e, i.e., $f \equiv q - q_e$. The greenhouse effect is related to the absorption of the surficial thermal radiation by greenhouse substances of the atmosphere.

The surficial thermal radiation is close to the radiation of a perfect black body with a surface temperature T, for which the spectral distribution with frequencies ω is determined by Planck's formula $I(\omega, T)$. In good approximation, the Stefan–Boltzmann formula σT^4 gives the total surface radiation after integrating Planck's distribution over all frequencies [1].

Dependence of the Greenhouse Effect on the Concentrations of Greenhouse Substances

Let us assume that the total number N of greenhouse gases with different concentrations overlaps the whole spectrum of the Earth's thermal radiation. Let us consider the existence of spectral windows assuming that the concentration of greenhouse gas, which absorbs thermal radiation in the relevant spectral interval, is equal to zero. Let us denote the portion of the surficial thermal radiation in the absorption band region ω_l of the lth greenhouse gas ($l = 1, 2, \ldots, N$) by $\Delta\delta_l$:

$$\delta_l = \Delta\omega_l I(\omega_l, T)/\sigma T^4, \quad \sum_{l=1}^{N} \delta_l = 1.$$

For each greenhouse gas, we can determine the thickness of the layer that absorbs the whole thermal radiation influx within the relevant spectral interval (within measurement error). Let us call this layer the optically dense layer of greenhouse gas and denote the number of optically dense layers of the lth greenhouse gas in the atmosphere by n_l. This number is proportional to the atmosphere's optical thickness, which is equal to the number of layers that attenuate the radiation influx by the factor $e = 2.718$ (see, for example, [2, 3]). Each optically dense layer of gas absorbs the whole radiation influx and then emits it in various directions, i.e., upward and downward in the unidimensional model (Fig. 1). Each layer only receives the radiation emitted by the neighboring upper and lower layers. The radiation of the rest of the layers falls short of the given layer, because it is completely absorbed by other layers. The number of optically dense layers is proportional to the gas concentration. If the absorption bands of several gases are coincident, the number of optically dense lay-

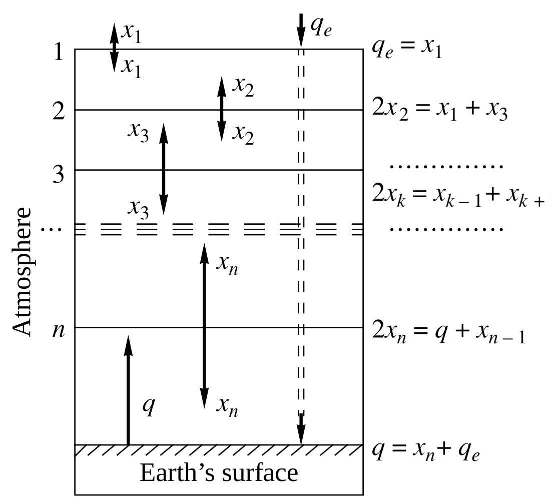

Fig. 1. Greenhouse effect vs. number of optically dense layers (n) for the case of a greenhouse gas, absorbing radiation over the whole spectral region of thermal radiation from the Earth's surface, $\delta_l = 1$. x_k ($k = 1, 2, 3, \ldots, n$) is the intensity of thermal radiation, emitted by the kth gas layer upward and downward; q is the intensity of the surficial thermal radiation; q_e is the intensity of the Sun's radiation absorbed by the Earth's surface and is equal to the intensity of thermal radiation emitted by the upper radiation layer x_1. It is assumed that the atmosphere is transparent for the Sun's radiation.

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ers with the given absorption band will be equal to the sum of numbers of optically dense layers for all these gases.

Denoting the upward radiation flux of the kth layer of greenhouse gas (and equal downward radiation flux) by $x_{l,k}$ (l is the greenhouse gas number, k is the layer number, $x_{l,1}$ is the upward flux from the upper radiation layer), we obtain the following system of linking equations for the energy balance (conservation) in each layer for each greenhouse gas:

$$q_e = \sum_{l=1}^{N} x_{l,1}, \quad 2x_{l,1} = x_{l,2},$$
$$2x_{l,k} = x_{l,k-1} + x_{l,k+1,\dots}, \tag{1}$$
$$2x_{l,n_l} = x_{l,n_l-1} + q\delta_l, \quad q = q_e + \sum_{l=1}^{N} x_{l,n_l}.$$

Here, q and q_e are space-oriented thermal radiation fluxes from the Earth's surface and the atmosphere's upper radiation layer, respectively. The first and penultimate equations represent the boundary conditions in the atmosphere's upper radiation layer and on the Earth's surface. The last equation is a consequence of all the other equations. The solution of internal equations has the form

$$x_{l,k} = kx_{l,1}, \quad k = 1, 2, \dots, n_l. \tag{2}$$

Substituting this solution in the penultimate equation and solving it together with the first equation, we obtain

$$q = \frac{q_e}{b}; \quad q - q_e \equiv f = Bq;$$
$$B = 1 - b; \quad b = \sum_{l=1}^{N} \frac{\delta_l}{n_o + n_l + 1}, \tag{3}$$

where $b = 1 - B$ is the space-oriented portion of the surficial thermal radiation; B is the value of the normalized greenhouse effect (the portion of thermal radiation reflected to the surface by the atmosphere [3–5]); and n_o is the number of optically dense layers of cloudiness, which absorbs thermal radiation in all spectral ranges.

If the number of optically dense layers n_l is of the same order for all greenhouse gases, the intensity of the Earth's surface radiation q and the greenhouse effect f grow with increasing n_l. In this case, the coefficient of reflection B tends to unity and the transmission factor b approaches zero.

The lifetime of molecular excitations of greenhouse gases is superior to the time intervals between successive collisions of air molecules [4]. Therefore, we can assume that the excitation energy of greenhouse gas molecules in each optically dense layer is approximately evenly distributed over all degrees of freedom of the air molecules, and the temperature based on the brightness [3, 4] of the greenhouse gas radiation bands in each layer approximately coincides with the temperature of the air in this layer.

The numbers n_l are different for different greenhouse gases. Under slight cloudiness, $n_o \ll n_l$ and the upper radiation layers of different absorption bands with various frequencies ω are localized at different altitudes and are characterized by different values of the effective brightness-related temperature. However, the difference between the frequency-dependent brightness-related temperatures and between the positions of the upper radiation layer is small at $n_l \gg 1$. Since the difference between temperatures of the Earth's surface and the upper radiation layer is small, relative to the absolute surface temperature, the solution (2) of Eq. (1) corresponds to the fact that the temperature of each atmospheric layer is inversely proportional to its altitude (according to observations in [3, 5]).

In the Earth's atmosphere, the main greenhouse gases are water vapors and CO_2. The spectral window of the clear sky is closed under cloudy skies. The value b (3) of the Earth's atmosphere can be written in the form

$$b = \frac{\delta_{H_2O}}{n_o + n_{H_2O} + 1} + \frac{\delta_{CO_2}}{n_o + n_{CO_2} + 1} + \frac{\delta_o}{n_o + 1}, \tag{4}$$

where δ_o is the portion of thermal radiation going out through the spectral window closed by cloudiness. It is easy to verify that b monotonically decreases in (3) and (4), while f monotonically increases in (3) with increasing n_o, n_{H_2O}, and n_{CO_2}. Note that the CO_2 absorption band and, accordingly, δ_{CO_2} in b (4) increase logarithmically with increasing CO_2 concentration [1, 6]. However, Eq. (4) contains the explicit n_{CO_2} dependence ignored in traditional calculations [6]. Based on (3) and (4), the greenhouse effect increases linearly with increasing cloudiness at $n_o \gg n_{H_2O}$ and $n_o \gg n_{CO_2}$:

$$f = q_e\{n_o + \delta_{H_2O}n_{H_2O} + \delta_{CO_2}n_{CO_2}\}. \tag{5}$$

Based on available observations, the Venusian greenhouse effect amounts to 99% (table). Hence, practically all thermal radiation of the Venusian surface is completely absorbed by its atmosphere and is reflected to the surface.

The Venusian atmosphere consists of a great number of optically dense layers in all spectral regions of the surficial thermal radiation. Based on the above estimations, the greenhouse effect grows with the concentration of substances in the Venusian atmosphere. A similar effect should be induced by the intensification of cloudiness in the Earth's atmosphere with increasing surface temperature.

Thermal characteristics of planets

Planets	Sun's constant	$A = B = 0$ Orbital temperature	$A > 0, B = 0$ Thermal radiation into the space		$A > 0, B > 0$ Average values on the surface of planets		
	I, W/m^2	t, °C	A, %	t, °C	B, %	t, °C	p, atm
Mars	589	–48	15	–56	7	–53	0.01
Venus	2613	+58	75	–41	99	+460	93
Earth 2	1367	+5	30	–18	40	+15	1
Earth during complete glaciation 1	1367	+5	80	–90	7	–85	1
Earth during complete evaporation of oceans 3	1367	+5	75	–80	99	+400	300

Note: I (Sun's constant) is the power of the thermal radiation flux beyond the atmosphere; A (planetary albedo) is the relative reflection of the thermal radiation by the Earth; B (normalized greenhouse effect) is the relative reflection of the Earth's thermal radiation to the surface, and p is the atmospheric pressure. Under the Earth's complete glaciation (state 1), its albedo is assumed to be equal to the albedo of the glacier sheet and snow cover, while the coefficient of greenhouse effect is adopted to be equal to the coefficient of the Martian greenhouse effect. The state of the Earth's complete glaciation (state 1) is below the melting point of CO_2 (–78.5°C). Under the complete evaporation of oceans, the greenhouse effect and albedo are adopted to be equal to relevant values on Venus (state 3). On Venus, whose atmosphere consists of 96% CO_2, this gas is over the critical point, which is defined for CO_2 by temperature of 31°C and pressure of 73 atm. State 3 of the complete evaporation of the Earth's hydrosphere is also over the critical point for water (374°C, 219 atm) [1, 7, 8].

Nature of the Earth's Climatic Stability

The global mean energy balance on the Earth's surface, including the atmosphere, has the form

$$C\frac{dT}{dt} = q_{\text{in}} - q_{\text{out}} \equiv -\frac{dU}{dT}, \quad (6)$$

$$q_{\text{in}} \equiv \frac{I}{4}a, \quad q_{\text{out}} \equiv q_e \equiv qb, \quad q = q_e + Bq = \sigma T^4, \quad (7)$$

where T is the absolute temperature of the Earth's surface; C is the mean heat capacity per unit surface area; $I/4$ is the Sun's radiation flux; I is the Sun's constant (Sun's radiation flow per unit of the Earth's cross-sectional area); $a = 1 - A$ is the portion of the Sun's radiation absorbed by the Earth; A is the albedo (portion of the Sun's reflected radiation); values q, q_e, b, and B are defined above; and $U(T)$ is the potential function (Lyapunov's function). The single independent variable of the balance Eq. (6) is the temperature T.

In the stationary state of the zero-order energy increment rate, $C\frac{dT}{dt} = 0$, the derivative of potential function $U(T)$ with respect to temperature turns to zero, and the function $U(T)$ has an extremum (maximum or minimum). In this case, the central part of Eq. (6) defines the value of the stationary temperature $T = T_s$:

$$\frac{I}{4}a(T_s) - \sigma T^4 b(T_s) = 0$$

or

$$T_s = T_0\left(\frac{a(T)}{b(T)}\right)^{1/4}, \quad T_0 \equiv \left(\frac{I}{4\sigma}\right)^{1/4} = 278\text{ K}. \quad (8)$$

The second derivative $W \equiv \left.\frac{d^2U}{dT^2}\right|_{T = T_s}$ defines the character of the extremum. The extremum is a stable minimum at $W > 0$ and an unstable maximum at $W < 0$.

$$W \equiv \left(\frac{d^2U}{dT^2}\right)_{T = T_s} = \frac{I}{4}\left(\frac{a(T)}{T}(4 + \beta - \alpha)\right)_{T = T_s}, \quad \alpha \equiv \frac{da}{dT}\frac{T}{a}, \quad \beta \equiv \frac{db}{dT}\frac{T}{b}. \quad (9)$$

The stationary state is stable for $\alpha - \beta < 4$ and unstable for $\alpha - \beta > 4$.

The solution of equality for the stationary value $T = T_s$ (8) can be obtained by plotting the curve $y_1(T) = T_0\left(\frac{a(T)}{b(T)}\right)^{1/4}$ versus T and finding the points of intersection of this curve and the straight line $y_2(T) = T$. Derivatives from the functions $y_1(T)$ and $y_2(T)$ with respect to temperature have the form

$$y'_1\big|_{T = T_s} = \frac{\alpha - \beta}{4}, \quad y'_2 = 1. \quad (10)$$

The condition of stability $\alpha - \beta < 4$ is equivalent to condition $y'_1\big|_{T = T_s} < 1$. The condition of instability $\alpha - \beta > 4$ is equivalent to $y'_1\big|_{T = T_s} > 1$.

The main part of the Earth's greenhouse effect is defined by the number of optically dense layers of water vapor n_{H_2O} [see Eq. (4)], which is proportional to the water vapor concentration. The latter, in turn, is proportional to the saturating concentration [5]. The

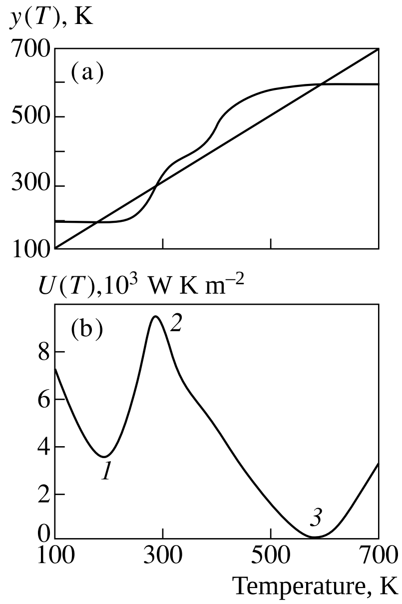

Fig. 2. Physical stability of possible climates of the Earth. (a) Graphical solution for the determination of stationary states *1, 2, 3* [see Eq (6)]. Intersections of curves correspond to extremums of potential function U (6). (b) Potential function U, describing the stability of the Earth's climates. Minimums correspond to stable states, while maximums correspond to unstable states.

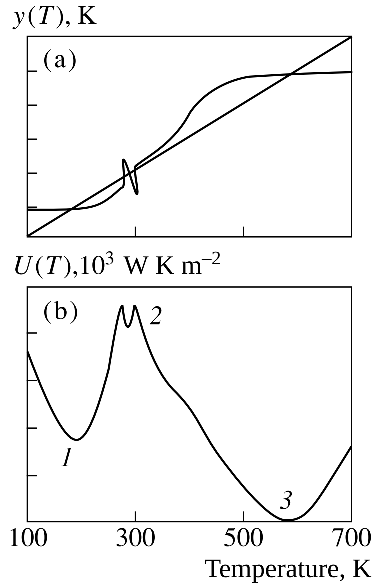

Fig. 3. Biotic stability of the Earth's recent climate (see caption of Fig. 2). To ensure stability of the recent climate, it is necessary that specific feature of the potential function U should be generated in the point corresponding to the recent global mean temperature. The appearance of this specific feature could be provided only due to governing action of global biota on the Earth's climate.

saturating concentration depends on temperature, according to the Clausius–Clapeyron law [5]:

$$n_{H_2O}(T) = ce^{-T_{H_2O}/T} \equiv e^{\sigma - T_{H_2O}/T},$$
$$T_{H_2O} \equiv \frac{Q_{H_2O}}{R} = 4871 \text{ K}, \tag{11}$$

where Q_{H_2O} is the latent evaporation heat for 1 mole of water vapor, R is the gas constant, T_{H_2O} is the effective barrier temperature of water evaporation, and $c \equiv e^{\sigma}$ is a temperature-independent constant. To plot function $b(T)$ (4), let us estimate the contributions of the spectral regions by the values $\delta_{H_2O} \approx 0.6$; $\delta_{CO_2} \approx 0.3$; $\delta_0 \approx 0.1$ [1, 3, 5]. At present, cloudiness determines about 23% of the greenhouse effect and about one-third of the greenhouse effect from water vapor. Assuming this relation to be conserved over a wide range of temperatures, let us assume that $n_0(T) = 0.3 n_{H_2O}(T)$. Today, the atmospheric CO_2 concentration is about 10% of the water vapor concentration, and the former is not perceptibly increased with increasing temperature. Therefore, let us assume $n_{CO_2} = 0.1 n_{H_2O}$ (288 K). Setting conditions of complete glaciation in the form $b(238 \text{ K} = -35°\text{C}) = 0.95$, we obtain $\sigma = 17.1$ for constant σ in (11). The consideration of finiteness for the hydrosphere can be made by stopping the growth of $n_{H_2O}(T)$ [see Eq. (11)] for $T = T_{cr}$, when $b(T_{cr}) = 0.01$ as on Venus (see table). For a given $b(T)$ and $a(T)$, the potential function U (6) is built unambiguously on the conditions of its continuity and equality to zero at the least minimum point (Fig. 2).

Under temperature changes ranging from complete glaciation to complete evaporation of the Earth's hydrosphere, the greenhouse effect and $b(T)$ change approximately one hundred times, whereas albedo and $a(T)$ values change only 2–3 times (see table). Therefore, the character of the $a(T)$ behavior, depending on the temperature setting in the form of a Gaussian function, has no practical effect on the solutions of Eq. (8) or on the character of their stability. As seen in Fig. 2, there are only two physically stable states: complete glaciation (*1*) and total evaporation of the hydrosphere (*2*). These states are divided by a physically unstable state corresponding to the maximum of potential function $U(T)$.

To obtain the real stable state of the Earth's recent climate, it is necessary that the features of functions $b(T)$ and $a(T)$ (Fig. 3) are generated in the liquid phase area of the hydrosphere. These features could be realized only due to the regulatory functioning of the global biota [9]. Notice that the biota not only changes the character of the extremum, but also slightly shifts its position. In Fig. 2, the physically unstable maximum coincides with the observed stable minimum ($T_{min} = T_{max} = 288$ K), providing that complete glaciation occurs at $T = 247$ K (–26°C). If we assume that complete glaciation corresponds to $b = 0.95$ and sets in at

T_{max} > 247 K (–26° C), the physical maximum will shift to the right (T_{max} > 288 K). In this case, the stable depression of the recent climate is located on the physical slope oriented toward complete glaciation.

Biotic governance of the Earth's climate is based on consumption of the Sun's radiation by biota (photosynthesis). Ordered environmental processes are generated by biota with the efficiency η_s, which is determined by the relative difference between temperatures of the Sun (T_S) and the Earth (T_E): $\eta_S = (T_S - T_E)/T_S \sim 0.95$. All processes of global circulation in the atmosphere and ocean are caused by a temperature difference at the equator and poles, which is on the order of $\Delta T_E \sim 30°C$. The efficiency of these processes $\eta_E = \Delta T_E/T_E \sim 0.1$ is an order of magnitude less than that of processes generated by biota. The powers of the energy absorbed by biota (including transpiration) and processes of global circulation are approximately equal [9]. Hence, the generation of ordered processes that affect the Earth's climate by global circulation is an order of magnitude less relative to the global biota, which completely determines the stability of the Earth's climate.

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