The first version of "No reactive motion arising from condensation" (http://www.bioticregulation.ru/ab.php?id=ban, http://arxiv.org/abs/1212.3100v1) was submitted to the Journal of the Atmospheric Sciences and rejected. The Editor encouraged the authors to resubmit the paper refuting the referees' comments:

"The two reviewers recommend rejection. They both point to fatal flaws in your argument, albeit from different perspectives. Perhaps after correcting the flaws, one can ask a question how important the term in question is in typical cloud conditions. Such a study can eventually lead to a publishable manuscript.?"

This was done. The complete current paper is here: http://www.bioticregulation.ru/ab.php?id=ban

Below there are two reviews followed by the authors' reply followed by the original submission.

Review
No reactive motion arising from condensation

A. M. Makarieva et al.

Recommendation Reject

Summary

This note disputes existing derivations of the momentum equation for a moist fluid flow undergoing condensation and condensate fallout. The authors' derivation is incorrect (see below) and therefore the claim made in this paper is without merit.

Newton's Second Law for a Continuum

Newton's Second Law of Motion,

$$\frac{d(m\mathbf{u})}{dt} = \mathbf{F}_{ext} \quad , \tag{R1}$$

refers to a particle with mass m and velocity $\mathbf{u}$. In words, the time rate of change of the particle's momentum ($m\mathbf{u}$) equals the external applied force $\mathbf{F}_{ext}$. In a continuum of particles the momentum is defined by

$$\int \rho \mathbf{u} d\tau \tag{R2}$$

(Batchelor, 1967, p. 137) where ρ is the density and $d\tau$ is an element of volume. The time rate of change of (R2) is (following Batchelor, 1967, p. 134)

$$\frac{d}{dt}\int \rho \mathbf{u} d\tau = \int \rho \frac{D\mathbf{u}}{Dt} d\tau + \int \mathbf{u} \left(\frac{D\rho}{Dt} + \rho(\nabla \cdot \mathbf{u}) \right) d\tau \quad . \tag{R3}$$

Under normal conditions, the second term on the rhs of (R3) vanishes and Newton's Second Law for a continuum takes the form of Eq (3.2.2) of Batchelor (1967).

In the present circumstances of moist airflow, the authors take their (4) as the starting point for the momentum equation of the mixture $\rho_m = \rho_a + \rho_v$ and with $\mathbf{F}_a$ describing "...the internal forces that describe the exchange of momentum between the droplets and the moist air." However in the present case there may be condensation and hence the second term on the rhs of (R3) does not vanish and the proper momentum equation is

$$\rho_m \frac{D\mathbf{u}_a}{Dt} + \dot{\rho}_v \mathbf{u}_a = -\nabla p + \nabla \cdot \sigma + \rho_m \mathbf{g} + \mathbf{F}_a \tag{R4}$$

which is the same as the authors' (4) except for the absence of the second term on the lhs in their equation. Analogously the momentum equation for the liquid is

$$\rho_l \frac{D\mathbf{u}_l}{Dt_l} - \dot{\rho}_v \mathbf{u}_l = \rho_l \mathbf{g} + \mathbf{F}_l \quad . \tag{R5}$$

As noted by the authors, $\mathbf{F}_a = -\mathbf{F}_l$, and therefore the combination of (R4) and (R5) gives

$$\rho_m \frac{D\mathbf{u}_a}{Dt} + \rho_l \frac{D\mathbf{u}_l}{Dt_l} = -\nabla p + \nabla \cdot \sigma + \rho_m \mathbf{g} + \rho_l \mathbf{g} + \dot{\rho}_v \mathbf{v}_l \tag{R6}$$

which is identical to the authors' (9) and which is said to be the incorrect result. However since the authors begin with the incorrect momentum equations [their (4) and (6)], they arrive at an incorrect result [their (9) or (R6) without the last term]. One final point is that it would be incorrect to lump the condensation terms, which form part of the time rate of change of momentum, in with the physical forces ($\mathbf{F}_a, \mathbf{F}_l$).

Note that the foregoing derivation never uses the conservation law given by the authors' (8) which is said to be the root of the problem with Bannon's (2002) derivation.

Comments on "No reactive motion arising from condensation" by Anastassia M. Makarieva, Victor G. Gorshkov, Andrei V. Nefiodov, Douglas Sheil, Antonio Donato Nobre, Peter Bunyard, and Bai-Lian Li

Recommendation: Unacceptable

General Comments: This manuscript argues that the article by Bannon (2002), “Theoretical foundations for models of moist convection”, (henceforth B02) contains an error in the momentum equation for moist air. Such a contribution would be worthy of publication in *JAS*. However the argument is logically flawed and without merit. Publication can not be recommended.

Major Comments

1. The authors correctly note that the momentum equation (5.18) of B02 contains the terms on the right hand side of the equation

$$\rho_l \mathbf{g} + \dot{\rho}_v \mathbf{v}_l \tag{1'}$$

The second term is the reactive term that they question. However they incorrectly label this reactive term a force in their (1). In contrast B02 does not label the term a force. They proceed to provide insight by deriving the reactive term in section 2. Their derivation in (2) and (3) correctly invokes the conservation of momentum *in the absence of forces* to derive the reactive term. Thus they are logically inconsistent in labeling the reactive term as a force. The reactive term can not simultaneously be a force and be derivable from an argument assuming momentum conservation and hence the absence of forces.

2. B02 does not claim that (1’) is a force. Thus their (5) is invalid and their argument in the paragraph starting on line 92 that there is a contradiction in B02 is invalid.

3. The authors note no math errors in the analysis of the momentum equation in B02. The reactive term is also present in the multiphase momentum equations of Crowe et al (1998) *Multiphase Flows with Droplets and Particles*.

List of revisions made in response to the referees' comments

1. In a separate section, we have considered the alternative derivation of the reactive motion term proposed by Referee 1. We have shown that this derivation is based on an incorrect application of Newton's second law to systems of variable mass. This misunderstanding was discussed in considerable detail in the literature, but predominantly outside the meteorology domain. We believe that it would be valuable to alert JAS readers to this (non-trivial) physical problem.

2. Referee 1 quoted the capital volume by Batchelor (1967). In the revised manuscript we show that Batchelor in some way anticipated the problem with the derivations similar to that of Bannon (2002). He mentioned that the source function (dotted chi) needs to be defined dependent on the physical nature of the considered variable chi (total momentum in our case). The neglect of this problem by Bannon (2002) was the cause of the error.

3. Referee 2 major objection was that *"Thus they are logically inconsistent in labeling the reactive term as a force. The reactive term can not simultaneously be a force and be derivable from an argument assuming momentum conservation and hence the absence of forces"*.
   We note that momentum conservation does not assume the absence of forces, it assumes the absence of *external* forces. Calling a term in the dynamic equation of motion "force" or "rate of momentum transfer" is a matter of notation, not essence. However, responding to the concerns of Referee 2, in the revised manuscript we never refer to the reactive motion term as "a force". This has not affected any of our results.

4. Referee 2 also stated that we did not find math errors in the derivations of Bannon (2002). The error in the derivation of Bannon (2002) consists in the fact that the derivation started from an incorrect modification of fundamental physical equations. Starting from an incorrect physical equation it is possible to obtain an incorrect physical result without making mathematical errors.

5. Finally, in the revised manuscript we clarified that our analysis as presented in the original version resolves the long-standing contradiction between the derivations of Ooyama (2001) – which do not contain the reactive motion term – and those of Bannon (2002). Indeed, the reactive motion term does not exist.

We thank the referees for useful comments.

No reactive motion arising from condensation

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ABSTRACT

Bannon (2002, J. Atmos. Sci. **59**: 1967–1982) developed expressions governing the motion of moist air on the basis that when droplets condense and begin to fall the remaining gas moves upwards so as to conserve momentum. Here we show that such a reactive motion is based on a misunderstanding of the conservation of momentum in the presence of gravitational field and does not exist.

1. Introduction

The foundations of meteorology and climate sciences lie in the physical principles that govern atmospheric behavior. Basic physical principles, such as the conservation of momentum, must be correctly understood and applied. Here we identify and explain a key error in one published fundamental equation governing atmospheric motion that has persisted in the literature for a decade.

Bannon (2002) investigated how the Navier-Stokes equations of air motion are modified by the presence of condensate particles in the atmosphere. He derived the following expression for the force $\mathbf{F}_a$ that acts on moist air in the presence of condensation:

$$\mathbf{F}_a = \rho_l \mathbf{g} + \mathbf{v}_l \dot{\rho}_v. \tag{1}$$

Here $\rho_l \equiv N_l \overline{m}$ is the density of condensate particles in the air, N_l is the concentration of condensate particles in the air, $\overline{m}$ is their mean mass, $\mathbf{g}$ is acceleration of gravity, ρ_v is water vapor density, $\dot{\rho}_v$ (kg m^{-3} s^{-1}) is condensation rate and $\mathbf{v}_l$ is the mean droplet velocity relative to the air. Without losing the generality of our conclusions we consider only liquid droplets here.

The second term in Eq. (1) describes an upward acceleration of air upon condensation. Bannon (2002, p. 1972) reasoned the need for this term as a "reactive motion": with the droplets acquiring a downward velocity, the remaining air must gain an upward velocity so that total momentum (air plus droplets) is conserved. Here we shall show that in the presence of gravitational field this upward acceleration contradicts physical principles and does not exist.

2. Misunderstanding the conservation of momentum

Consider the argument for the reactive motion term in Eq. (1). Bannon (2002, p. 1972) discusses a body of mass M that is subjected to no external forces. For an example of such an idealized system, we can take a rocket that moves with velocity $\mathbf{u}_a$ in the absence of gravity. The rocket's motion is reactive: it expels some of its parts that move at a different velocity $\mathbf{u}_l$. The mass of the rocket diminishes at a rate $dM/dt < 0$. Neglecting the acceleration of the expelled fuel/exhaust, $d\mathbf{u}_l/dt = 0$ (cf. the forthcoming Eq. (7)), we can write the momentum conservation equation for the rocket and all of its parts as

$$\frac{d(M\mathbf{u}_a)}{dt} - \mathbf{u}_l \frac{dM}{dt} = 0, \tag{2}$$

$$M\frac{d\mathbf{u}_a}{dt} = \mathbf{v}_l \frac{dM}{dt}, \tag{3}$$

where $\mathbf{v}_l \equiv \mathbf{u}_l - \mathbf{u}_a$. Thus, acceleration of the rocket is proportional to the rate at which it loses mass and to the difference $\mathbf{v}_l$ between the rocket velocity and the velocity of its expelled parts. Equation (3) describes the reactive motion. Its right-hand part is fully analogous to the second term in Eq. (1), while Eq. (3) is equivalent to Eq. (B5.16) ("B" refers to equations of Bannon (2002))[1].

However, unlike our hypothetical rocket, atmospheric air does not exist in a space devoid of external force – it is subjected to gravity. The conservation of momentum (3) applicable to our rocket+exhaust system is not applicable to the air alone. The Earth's gravitational field is not a detail here: it is central to the process. The droplets do not fall because of some localized internal force acting within air volume of which they were part (unlike the fuel/exhaust that is expelled from the rocket). The newly formed droplets accelerate freely in a uniform downward direction not under some subtle process of physical chemistry, but under the pervasive influence of the Earth's gravity. Certainly the momentum of the system is conserved but here "the system" includes the entire bulk of the planet Earth. Based on

[1]We note that the equation for reactive motion derived by Bannon (2002) was incorrect. He stated that Eq. (3), or (B5.16), is equivalent to $dw/dt = -(w/M)dM/dt$, where w is velocity of the body subjected to no forces. However, inspection of Eqs. (3) and (B5.16) reveals that there are *different* velocities $\mathbf{u}_a$ and $\mathbf{v}_l$ on each side of the equations. This is because the different parts of the body move at different velocities (e.g., the rocket and its fuel/exhaust). At $dM/dt \neq 0$ the only physically plausible solution of Bannon's single-body equation $dw/dt = -(w/M)dM/dt$ is $dw/dt = 0$ and $w = 0$.

these considerations we conclude that the proposed reactive motion of moist air based on the supposed conservation of momentum of the isolated system (air+droplets) has no physical basis. It does not exist.

3. Examining the derivations of Bannon (2002)

We first revise the assumptions used by Bannon (2002) to obtain Eq. (1). Bannon (2002, p. 1970) noted that the only external forces that act on a unit atmospheric volume that contains moist air and droplets are the total ambient pressure gradient force, frictional forces and gravity. The equation of motion for moist air that reflects these considerations can be written as

$$\rho_m \frac{D\mathbf{u}_a}{Dt} = -\nabla p + \nabla \cdot \sigma + \rho_m \mathbf{g} + \mathbf{F}_a, \quad \frac{D}{Dt} \equiv \frac{\partial}{\partial t} + (\mathbf{u}_a \cdot \nabla). \tag{4}$$

Here $\rho_m \equiv \rho_a + \rho_v$ is the density of moist air (excluding condensate), ρ_a is dry air density, ρ_v is water vapor density, p is air pressure, σ is the viscous stress tensor, and $\mathbf{u}_a$ is the air velocity. The first three terms in the right-hand side of Eq. (4) describe the three forces that are external in the terminology of Bannon (2002), while $\mathbf{F}_a$ stands for internal forces that describe the exchange of momentum between the droplets and the moist air. According to Bannon (2002, p. 1970), the internal forces that describe the interaction between the moist air and the droplets are equal in magnitude and opposite in direction:

$$\mathbf{F}_a = -\mathbf{F}_l. \tag{5}$$

Next, Bannon (2002) used the following equation of motion for droplets:

$$\rho_l \frac{D\mathbf{u}_l}{Dt_l} = \rho_l \mathbf{g} + \mathbf{F}_l, \quad \frac{D}{Dt_l} \equiv \frac{\partial}{\partial t} + (\mathbf{u}_l \cdot \nabla). \tag{6}$$

Here $\mathbf{u}_l$ is the mean velocity of droplets in a unit atmospheric volume and $\mathbf{F}_l$ is the force exerted by the air on the droplets. Equation (6) is equivalent to (B5.8a) neglecting the impact of the large-scale pressure gradient on the droplet (as also done by Bannon (2002,

p. 1972)). Force $\mathbf{F}_l$ (6) is, in the notations of (B5.8a), equal to $\mathbf{F}_l \equiv -\rho_l \mathbf{v}_l / \tau_{vl}$. Finally, Bannon (2002, (B5.11a)) observing that droplets fall at a nearly constant velocity for most of their lifetime proposed that

$$\frac{D\mathbf{u}_l}{Dt_l} = 0. \tag{7}$$

Using Eq. (7) in Eq. (6) we immediately obtain from Eq. (5) that $\mathbf{F}_a = \rho_l \mathbf{g}$.

So how could Bannon (2002) have obtained $\mathbf{F}_a = \rho_l \mathbf{g} + \mathbf{v}_l \dot{\rho}_v$ (1) using the same assumptions (4)-(7)? The problem consists in the key equation (B2.2) of Bannon (2002):

$$\frac{\partial \chi}{\partial t} + \nabla \cdot (\chi \mathbf{u}_\chi) = \dot{\chi}. \tag{8}$$

This equation represents a general conservation relationship for an arbitrary property χ that is transported with velocity $\mathbf{u}_\chi$. Here $\dot{\chi}$ is described by Bannon (2002) as the rate at which property χ is being created within the considered unit volume. A problem with Eq. (8) is that $\dot{\chi}$ remains undefined. For any property χ, one must refer to some relationship (here a physical law) to determine $\dot{\chi}$. Bannon (2002) stated that writing Eq. (8) for the total momentum for a unit atmospheric volume, i.e. putting $\chi \equiv \rho_m \mathbf{u}_a + \rho_l \mathbf{u}_l$, produces the following relationship, see (B5.2) and (B5.7):

$$\rho_m \frac{D\mathbf{u}_a}{Dt} + \rho_l \frac{D\mathbf{u}_l}{Dt_l} = -\nabla p + \nabla \cdot \sigma + \rho_m \mathbf{g} + \rho_l \mathbf{g} + \mathbf{v}_l \dot{\rho}_v. \tag{9}$$

Equation (9) implicitly defines $\dot{\chi}$ for total momentum per unit volume. Combining Eq. (9) with Eqs. (7) and (4) yields Eq. (1). (Equation (4) with $\mathbf{F}_a$ (1) is equivalent to Eq. (B5.18). In the notations of (B5.18) we have $\dot{\rho}_v \equiv -\rho_a \dot{r}_{\text{cond}}$ and $\mathbf{F}_a = \rho_a \dot{\mathbf{u}} \equiv \rho_l \mathbf{g} - \rho_a \dot{\mathbf{u}}_{\text{phase}} \equiv \rho_l \mathbf{g} + \mathbf{v}_l \dot{\rho}_v$, see (B5.18) and (B5.7b).)

At the same time, by summing Eqs. (6) and (4) and taking into account Eqs. (5) and (7), we find that in the resulting equation the reactive motion term – the last term in Eq. (9) – is absent. This contradiction between Eqs. (4)-(7) and (9) results from the fact that Eqs. (9) and (1) were not derived by Bannon (2002) from consistent fundamental equations, but stemmed from an apparently arbitrary definition of $\dot{\chi}$ to which an assumed "reactive motion term" $\mathbf{v}_l\dot{\rho}_v$ was added. Indeed, Bannon (2002) never explicitly defined $\dot{\chi}$. The resulting violation of the momentum conservation law illustrates that arbitrary modifications to basic physical relationships tend to yield results that contravene physical laws.

4. Conclusions

Recently Cotton et al. (2011) reflected upon the fundamental equations describing moist atmospheric dynamics. They concluded that the reactive motion term described by Bannon (2002) was of potential importance and warranted further study. Cotton et al. (2011) offered various graphical illustrations explaining how the phenomenon is manifested. Here we have shown that this reactive motion term was based on a physical error. It does not exist.

REFERENCES

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