Potential energy of atmospheric water vapor and the air motions induced by water vapor condensation on different spatial scales

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Abstract

Basic physical principles are considered that are responsible for the origin of dynamic air flow upon condensation of water vapor, the partial pressure of which represents a store of potential energy in the atmosphere of Earth. Quantitative characteristics of such flow are presented for several spatial scales. It is shown that maximum condensation-induced velocities reach 160 m s^{-1} and are realized in compact circulation patterns like tornadoes.

1 Introduction

Atmospheric air on Earth conforms to the ideal gas law with a high accuracy. The main physical property of the ideal gas is that its equation of state does not depend on molar masses of the gas mixture constituents. At fixed temperature, a given value of pressure can be obtained for a mixture with arbitrary molar masses by setting the value of molar density of the gas. The second important property of atmospheric air is the presence of a constituent that undergoes condensation under terrestrial temperatures and pressures – water vapor. Molar density of the moist air mixture is equal to the sum of molar densities of the dry air and water vapor.

In a motionless atmosphere (with the impact of the greenhouse substances neglected) the air temperature would be the same at all heights. Vertical distributions of all gases would follow the hydrostatic Boltzmann's distribution according to their molar masses. The exponential scale height of water vapor with molar mass 18 g mol^{-1} would be about 13 km, while the scale height of the major air constituents, nitrogen and oxygen, with similar molar masses $\sim$ 30 g mol^{-1} would have scale heights of approximately 8 km. In this case water vapor would be saturated at the surface only (due to the contact with the liquid hydrosphere) and had an undersaturated concentration elsewhere. There would be no evaporation or condensation in such an atmosphere.

However, irrespective of the presence or absence of the greenhouse substances, such a hypothetical distribution of vertically isothermal moist air appears to be unstable. Any fluctuation leading to an upward displacement of an air volume results in adiabatic cooling of the rising air. Air temperature drops such that the equilibrium water vapor concentration dictated by Boltzmann's distribution becomes oversaturated at all heights where the air ascends. This causes the water vapor to condense. Its concentration decreases down to the saturated concentration. Condensation diminishes the total air pressure and disturbs Boltzmann's distribution of moist air. The vertical gradient of air pressure becomes greater than the weight of a unit air volume. There appears an upward-directed force acting on a unit air volume. Static equilibrium of moist air in the gravitational field is no longer possible. There appears a rising flow of air masses induced by condensation. The process of condensation is sustained by continuous evaporation of water vapor from the hydrosphere. The upward-directed force that acts on moist air causing it to rise adiabatically was termed the evaporative-condensational force (Gorshkov and Makarieva, 2006; Makarieva and Gorshkov, 2007, 2009a).

Most part of the condensed water vapor leaves the atmosphere via precipitation. A minor part is maintained in the atmosphere by the rising air flow. This imposes a drag force on the rising air flow and leads to a reduction of the vertical velocity. However, unlike the flow of water vapor which condenses as it rises, the flow of the dry air components which conserved their mass, cannot be unidimensional (vertical). There inevitably appear horizontal legs in the condensation-induced air circulation. Condensation of water vapor in the ascending air produces both vertical and horizontal pressure gradients. Therefore, the presence of water vapor in the atmosphere contacting with a liquid hydrosphere leads to the formation of three-dimensional circulation patterns.

In the following sections we derive relations between the vertical and horizontal components of air velocities and condensation-induced pressure gradients. We further use the obtained results to describe large-scale circulation with approximately constant velocities when the pressure gradient force and the turbulent friction force coincide. We also apply these relations to describe hurricanes and tornadoes, where the pressure gradient forces appear to significantly exceed the turbulent friction forces. So far the condensation-induced air motions have not received a consideration in meteorology. Several relevant observations regarding the conventional approaches are made in the footnotes to the main text.

2 Continuity equation for moist air

The equations of state for moist air as a whole, as well as for its components – dry air and water vapor, include one and the same universal molar gas constant R and do not depend on molar masses and mass densities of the components[1]:

$$p = NRT, \;\; p_v = N_v RT, \;\; p_d = N_d RT, \tag{1}$$

where p, N, p_v, N_v, p_d, N_d are the pressure and molar density of moist air as a whole, water vapor and dry air, respectively.

In a circulating atmosphere where no condensation takes place, the ratios of molar densities of all components at all heights affected by the are equal to their mean atmospheric values. The reason is that the diffusional velocities that would restore Boltzmann's distributions depending on molar densities and molar masses of the components, are small compared to the dynamic velocities of the air flow. According to observations, mixing ratios of the non-condensable air constituents and the molar mass of dry air are the same at all heights in the troposphere.

In the absence of condensation the ratio $\gamma = N_v/N$ should not change at any changes of pressure and temperature. Consequently,

[1]Note that this fundamental universality of ideal gas is masked in the meteorological literature by the common usage of mass densities $\rho = MN$, $\rho_v = M_v N$ and $\rho_d = M_d N$ along with mass gass constants $R_{air} \equiv R/M$, $R_v \equiv R/M_v$ and $R_d \equiv R/M_d$. Usually $R_{air} \approx R_d$ is denoted as R, while the universal molar gas constant is practically never used (e.g., Glickman, 2000).

the process of condensation should be reflected in the changes of ratio γ. The molar rate of condensation per unit volume (mole $\mathrm{m}^{-3}\,\mathrm{s}^{-1}$) that is caused by the decrease of air temperature with height z due to the adiabatic ascent of moist air with vertical velocity w, is equal to

$$wN\frac{\partial\gamma}{\partial z} = w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right), \tag{2}$$

$$\gamma \equiv \frac{N_v}{N} = \frac{p_v}{p}, \quad \frac{d\gamma}{\gamma} = \frac{dN_v}{N_v} - \frac{dN}{N} = \frac{dp_v}{p_v} - \frac{dp}{p}. \tag{3}$$

The meaning of Eq. (2) is physically transparent. Condensation rate is determined by the change of water vapor concentration minus the change of total air concentration that is not related to condensation. We emphasize that the second term in brackets in Eq. (2) comprises the relative change of molar density N of moist air as a whole rather than molar density N_d of its dry component. This reflects the fact that restoration of equilibrium pressure distribution upon condensation affects the air mixture as a whole, including the remaining water vapor.

Since the condensation rate (2) is a function of molar (not mass) densities, it is convenient and physically transparent to write the continuity equation for ideal gas in terms of molar densities as well. In Cartesian coordinates (assuming that there is no dependence of the flow on y) the continuity equation takes the form, see Eq. (2):

$$\frac{\partial Nu}{\partial x} + \frac{\partial Nw}{\partial z} = w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right). \tag{4}$$

Taking into account that $N = N_d + N_v$ and

$$\frac{\partial N_d u}{\partial x} + \frac{\partial N_d w}{\partial z} = 0, \tag{5}$$

we have from Eq. (4):

$$\frac{\partial N_v u}{\partial x} + \frac{\partial N_v w}{\partial z} = w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right). \tag{6}$$

Expanding the derivatives in Eq. (6) and multiplying both parts of the equation by N/N_v we obtain

$$u\frac{N}{N_v}\frac{\partial N_v}{\partial x} + w\frac{\partial N}{\partial z} + N\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) = 0. \tag{7}$$

Now expanding the derivatives in the left hand part of Eq. (4) and using Eq. (7) we obtain

$$u\left(\frac{\partial N}{\partial x} - \frac{N}{N_v}\frac{\partial N_v}{\partial x}\right) = w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right). \tag{8}$$

Using definition (3) we can re-write the continuity equation (8) as

$$u\frac{\partial \ln \gamma}{\partial x} = -w\frac{\partial \gamma}{\partial z}. \tag{9}$$

Now, turning from molar densities N and N_v to pressures p and p_v (1) we can see that temperature dependencies cancel from Eq. (9) and the latter becomes[2]

$$u\left(\frac{\partial p}{\partial x} - \frac{1}{\gamma}\frac{\partial p_v}{\partial x}\right) = wp\gamma\left(\frac{1}{p_v}\frac{\partial p_v}{\partial z} - \frac{1}{p}\frac{\partial p}{\partial z}\right). \tag{10}$$

According to Clausius-Clapeyron equation, saturated pressure p_v of water vapor depends on temperature only. When the considered area is horizontally isothermal (temperature T does not depend on x) we have $\partial p_v/\partial x = 0$. This condition presumes the existence of an inflow of water vapor caused by local evaporation from the hydrosphere. This inflow partly compensates condensation that occurs in the atmospheric column[3]. In this case Eq. (10) takes the form

$$-\frac{\partial p}{\partial x} = \frac{w}{u}\frac{\Delta p}{h_\gamma}, \quad \Delta p \equiv p\gamma = p_v, \quad h_\gamma^{-1} \equiv h_v^{-1} - h^{-1}, \tag{11}$$

$$h_v^{-1} \equiv -\frac{1}{p_v}\frac{\partial p_v}{\partial z}, \quad h^{-1} \equiv -\frac{1}{p}\frac{\partial p}{\partial z}, \quad \frac{\partial p_v}{\partial x} = 0. \tag{12}$$

[2]Equation (4) is equivalent to the sum of Eqs. (5) and (6). If there is no condensation, the right-hand part of Eq. (4) is zero. Equations (4), (5) and (6) can then be re-written in terms of mass densities $\rho = N/M$, $\rho_d = N_d/M_d$ and $\rho_v = N/M_v$. Here M_v and M_d are constant (M_d is constant in agreement with observations because the molar ratios of the dry air constituents do not change with height). But the molar mass of moist air $M = \gamma M_v + (1-\gamma)M_d = M_d(1-0.38\gamma)$ depends on γ and, consequently, on z. Therefore, if the condensation rate in the right-hand part of Eq. (4) is not zero, it will change substantially upon transition from molar to mass densities. Specifically, if condensation rate is written in form of the right hand part of Eq. (2) with N and N_v changed to ρ and ρ_v, respectively, this will lead to the appearance of an incorrect multiplier M_v/M in the right hand part of Eq. (8). This would contradict the physical meaning of the ideal gas equations of state (1).

[3]It is assumed in the conventional meteorology that the cause of atmospheric circulation is the fact that the surface is not horizontally isothermal due to external differential heating. The condensation-induced circulation, in contrast, arises on a horizontally isothermal surface (although this is not an indispensable condition). Horizontal temperature inhomogeneities that can be observed after the circulation has established are the consequences of the horizontal inhomogeneity of the process of water vapor condensation.

All magnitudes entering (11) and (12) depend on x and z. Height h_γ has the meaning of characteristic height where all water vapor condenses. Heights h_v and h are the scale heights of water vapor and moist air, respectively.

In a large-scale stationary circulation, where constant friction forces compensate the equally constant pressure gradient forces (Makarieva and Gorshkov, 2009a), horizontal velocity u does not change with x. Velocity w should be understood as the vertical velocity averaged over height h_γ. The flux of air enters the circulation area horizontally via a vertical cross-section of area Dh_γ and leaves the circulation area vertically across a horizontal cross-section of area DL. Here L is the horizontal dimension (length) of the circulation area counted along the x-axis, D is the circulation width counted along the y-axis perpendicular to the horizontal air flow. Taking into account that the number of air mols n_{in} that enter the circulation area differ from the number n_{out} of air mols leaving the circulation area by a relatively small number of mols of condensed water vapor, $n_{in} - n_{out} \sim \gamma n_{in} \ll n_{in}$, to the accuracy of $\gamma \ll 1$ we can write (see also footnote 6 below):

$$Dh_\gamma u = DLw, \quad \frac{w}{u} = \frac{h_\gamma}{L}. \tag{13}$$

Putting (13) into (11) we obtain

$$-\frac{\partial p}{\partial x} = \frac{\Delta p}{L}, \quad \Delta p \equiv p\gamma = p_v. \tag{14}$$

Thus, total horizontal pressure drop is equal to partial pressure of water vapor.

To arrive to Eq. (14) three equations have been used: Eq. (6) for N_v, equation $\partial p_v/\partial x = 0$, see (12), that reflects that the circulation area is horizontally isothermal and that water vapor is saturated, and Eq. (5) for molar density N_d. The latter equation does not include the condensation rate term (2), which is present in (4) and (6). Equation (13) namely arises from Eq. (5) for $N_d \gg N_v$.

It follows from Eq. (14) that $p_v = \Delta p$ represents a store of potential energy, which is converted to the kinetic energy of moving air masses as the water vapor condenses[4]. By integrating (14) we

[4]In the absence of friction at constant pressure gradient the horizontal velocity grows with distance x as prescribed by Bernoulli's equation: $\rho u^2/2 = \Delta p = \gamma p$. Due to $\gamma \ll 1$ we observe

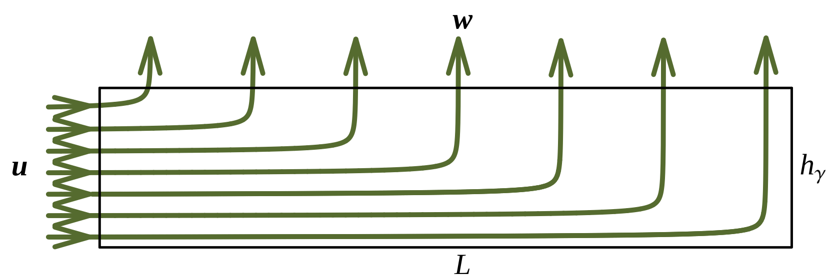

Figure 1: Air streamlines for a large-scale circulation of length L and height h_γ. The continuity equation (13) is $uh_\gamma = wL$.

obtain the following expression for potential energy $P(x) = p(x)$ that is defined to the accuracy of a constant term:

$$P(x) = p(x) = p(0) + \frac{\Delta p}{L}x. \qquad (15)$$

In the incoming air flow at $x = L$ water vapor is present everywhere in that part of the atmospheric column that is affected by the circulation. At $x = 0$ this water vapor has been completely used up and is partially replaced by locally evaporated water vapor.

Strictly speaking, Eqs. (13) and (14) are valid for a bunch of streamlines that enter the circulation area in the horizontal direction and leave it in the vertical direction. The number of horizontal streamlines present in the atmospheric column at a given x decreases as one travels inside the area at the expense of those streamlines that have left the area in the vertical direction. Velocitites u and w (13) and pressure gradient (14) are the same for each streamline;

that Δp is much smaller than p. On the other hand, we have $\Delta p/p = \gamma = \Delta\rho/\rho$. Mass density ρ changes little over the distance where air pressure changes by Δp: $\Delta\rho = \gamma\rho \ll \rho$. From this in the meteorological literature it is concluded that changes in density ρ due to condensation can be neglected (e.g., Sabato, 2008). . The resulting physical inconsistency is masked by the fact that changes of ρ and the formation of Δp are conventionally ascribed to independent physical causes. In particular, it is common in circulation models to take pressure profiles from observations. The relative smallness of horizontal pressure drop Δp is never discussed.

But if we put $\Delta\rho/\rho = \Delta p/p$ equal to zero, no velocity can form and the circulation cannot exist. This statement is general and does not depend on why Δp and $\Delta\rho = \rho_0 - \rho$ actually form. Mathematically, the error can be spotted as follows. From the equation of state (1) we have $p = C\rho$, where $C \equiv RT/M =$ const. for the considered horizontally isothermal surface. From Bernoulli's equation we then have $u^2 = 2\Delta p/\rho = 2C\Delta\rho/\rho = 2C(\Delta\rho/\rho_0)(1+\Delta\rho/\rho_0+...)$. As one can see, discarding $\Delta\rho$ compared to ρ does indeed correspond to discarding the term of a higher order of smallness. But with respect to the pressure gradient, the main effect is proportional $\Delta\rho$, which is the term of the first order of smallness. If the smallness is set to zero, the effect disappears.

the mean horizontal velocity in the column linearly decreases, while the height of ascent linearly grows, Fig. 1. All the above forms of the continuity equation, Eqs. (11)-(14), describe the kinematics of the air flow at given values of u and w. These velocities should be determined from Euler's equations with an account of friction[5]. Thus, condensation occurs as the moist air ascends in the vertical direction and is maintained due to evaporation from the horizontal Earth's surface. The associated condensational-evaporative force induces makes the moist air masses circulate along the streamlines that include both vertical and horizontal regions.

3 Condensation in the adiabatically ascending air

According to Clausius-Clapeyron equation, we have

$$\begin{aligned} \frac{dp_v}{p_v} &= \xi \frac{dT}{T}, \quad p_v(T) = p_{v0} \exp(\xi_0 - \xi), \\ -\frac{1}{p_v}\frac{\partial p_v}{\partial z} &= \xi \frac{\Gamma}{T} \equiv h_v^{-1}, \quad \Gamma \equiv -\frac{\partial T}{\partial z}, \quad \xi \equiv \frac{L_v}{RT}, \quad \xi_0 \equiv \frac{L_v}{RT_0}. \end{aligned} \tag{16}$$

where $L_v = 45$ kJ mol^{-1} is the molar heat of vaporization (latent heat).

[5]The major friction force per unit area of the Earth's surface, which opposes the pressure gradient force that would otherwise accelerate the air, is the friction force that can be called gravitational, as it is proportional to the weight of atmospheric column $\mu\rho g h = \mu p_s$, $\mu = z_T/h \sim 10^{-5}$. Here p_s is surface pressure, $z_T \sim 0.1$ m is the surface roughness (it is proportional to the height of vegetation cover, oceanic waves etc.), $h \sim 10$ km is the scale height of the atmosphere. The same form of gravitational friction, $\mu\rho g h$, is due to frictional dissipation of liquid drops precipitating or suspended in the atmosphere. In this case μ has the meaning of the relative volume occupied by the drops, $\mu \sim \gamma(w/w_b) \sim 10^{-5}$, where w_b is the mean downward velocity of the drops, $w/w_b \sim 10^{-3}$. The gravitational friction force does not depend on velocities u or w and can be represented as $\mu\rho g h = \rho u_{g*}^2$, where $\mu = u_{g*}^2/gh$ is Froude's number, u_{g*} has the meaning of rotation velocity for the turbulent eddies that bud from the main air flow due to gravitational friction. The gravitational friction force ρu_{g*}^2 exceeds by 30 times the force of aerodynamic friction $c_D \rho u^2 = \rho u_*^2$ ($u_{g*}^2 \sim 30 u_*^2$) that is usually taken into account in the Navier-Stokes equations and the formulation of Reynolds stress (Makarieva and Gorshkov, 2009a).

Moist air obeys the hydrostatic equilibrium distribution[6]

$$-\frac{1}{p}\frac{\partial p}{\partial z} = h^{-1} = \frac{Mg}{RT}, \quad M = M_d(1 - 0.38\gamma). \tag{17}$$

Functions $\Gamma(z)$, $T(z)$ and $\gamma(z)$ can be found from the first law of thermodynamics under condition that the ascent is adiabatic, using the definition of γ (3) and the Clausius-Clapeyron equation (16):

$$c_p\frac{\partial T}{\partial z} - \frac{1}{N}\frac{\partial p}{\partial z} + L_v\frac{\partial \gamma}{\partial z} = 0; \quad -\frac{1}{\gamma}\frac{\partial \gamma}{\partial z} = -\frac{1}{p_v}\frac{\partial p_v}{\partial z} + \frac{1}{p}\frac{\partial p}{\partial z} = h_v^{-1} - h^{-1} \equiv \frac{1}{h_\gamma}. \tag{18}$$

The first equation in (18) is the first law of thermodynamics for an adiabatic process, with the third term describing condensation, see (2); $c_p = c_v + R = (7/2)R$ is the molar heat capacity at constant pressure.

System of equations (16)-(18) can be re-written in a closed form:

$$-\frac{\partial T}{\partial z} \equiv \Gamma(z) = \Gamma_d \frac{1 + \gamma\xi}{1 + \mu\gamma\xi^2}(1 - 0.38\gamma), \tag{19}$$

$$-\frac{1}{\gamma}\frac{\partial \gamma}{\partial z} = h_\gamma^{-1} = \xi\frac{\Gamma(z)}{T} - \frac{M_d(1 - 0.38\gamma)g}{RT}. \tag{20}$$

$$\Gamma_d \equiv \mu\frac{T}{h_d} = 9.8\,\mathrm{K\,km^{-1}}, \; h_d \equiv \frac{RT}{M_d g}, \; \mu \equiv \frac{R}{c_p}.$$

Here Γ_d is the value of $\Gamma(z)$ at $\gamma = 0$, i.e. it is the adiabatic lapse rate of air temperature for dry air[7]. Eq. (19) at $\gamma \ll 1$ coincides with the well-known expression for moist adiabatic lapse rate of air

[6]Equation (17) should be more appropriately referred to as the equation of *aerodynamic equilibrium in the gravitational field of Earth*. Indeed, this equilibrium does not correspond to a static Boltzmann's distribution of gases with different molar masses. It arises in the result of the air ascent which occurs with a sufficiently high velocity w, which is the same for all gases (including the remaining water vapor) despite their different molar masses. In the system of rest of the ascending air, Eq. (17) corresponds to a hydrostatic distribution of air with a constant molar mass M and departs from the latter only insignificantly due to the fact that $\gamma \ll 1$ decreases with height. Vertical turbulent flux of water vapor associated with evaporation from the surface leads to the fact that the vertical velocity of water vapor w_v in the ascending air flow is always larger than the vertical velocity of air w, $w_v = w + \Delta w_v$ (Makarieva and Gorshkov, 2007). At $\gamma \ll 1$ this does not change the flux of moist air as a whole, $Nw = N_d w + N_v w_v = Nw(1 + \gamma\Delta w_v/w) \approx Nw$. The ratio between vertical velocities of water vapor and air as a whole reaches its maximum value $\Delta w_v/w \approx 1$ in a large-scale circulation where w is small. In compact intense circulation where condensation rate greatly exceeds the local rate of evaporation, we have $\Delta w_v \ll w$ and $w_v \approx w$.

[7]In the meteorological literature one can find statements (e.g., Pöschl, 2009, p. S12436) that the release of latent heat L_v upon condensation warms the air, as well as that latent heat is a major source of energy for some types of circulation (e.g., see discussion and refer-

temperature (Glickman, 2000)[8]. Solutions of Eqs. (19), (20) are shown in Fig. 2.

Due to the large value of the ratio $L_v/R \equiv T_v \approx 5300$ K, the dimensionless ratio $\xi \equiv T_v/T$ is always much larger than unity. Therefore, at $\gamma \to 1$ (this would happen if at constant temperature the dry component were largely removed from the atmosphere and the water vapor partial pressure became the dominant contributor to total air pressure) the adiabatic lapse rate (19) ceases to depend on z and tends to $\Gamma \to (\Gamma_d)(0.62/\mu\xi) = 1.2$ K km^{-1}. Scale height h_v (12) of saturated water vapor tends to the hydrostatic equilibrium value $h_v = T/(\Gamma\xi) \to h_d/0.62 = h_{v\,stat} = RT/(M_v g) = 13.5$ km. The right-hand part of the last equality in Eq. (18) turns to zero; γ ceases to depend on z. The third term containing $\partial\gamma/\partial z$ in the equation of the second law of thermodynamics (18) vanishes. Condensation, evaporation and the condensation-induced air circulation all stop.

The transition to hydrostatic equilibrium during adiabatic ascent of moist air at $\gamma \to 1$ implies that the weight of saturated water vapor column of unit area at any height becomes equal to saturated vapor pressure at this height: the evaporative-condensational force disappears. Consequently, the adiabatic ascent of moist air can no longer be maintained by this force and cannot arise spontaneously. In this case, in contrast to the case of $\gamma \ll 1$, the stable stationary state of the atmosphere is not the state of self-sustained air circulation, but the the state of hydrostatic equilibrium of motionless air (water vapor) with constant air temperature at all heights.

The condensation-induced circulation of the modern atmosphere occurs at a global mean surface temperature that is optimal for life. This temperature fixes the saturated concentration of water vapor in accordance with the Clausius-Clapeyron equation (16). Therefore, the condensation-induced circulation only becomes possible due to the high concentration of atmospheric nitrogen, which can be called an air "ballast". Indeed, unlike CO_2, nitrogen is not a greenhouse

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ences in Makarieva et al., 2010a). In reality, according to the Clausius-Clapeyron equation, condensation occurs when there is an external cause of cooling (e.g., adiabatic ascent). Condensation cannot warm the air to a temperature higher than the air had prior to condensation. During condensaton, the water vapor concentration decreases, which, according to the Clausius-Clapeyron law, corresponds to a drop (not a rise) of air temperature.

[8]We emphasize that γ in Eqs. (19), (20) is defined as in Eq. (3). At $p_d \to 0$ we have $\gamma \to 1$ and $\partial\gamma/\partial z \to 0$. In the meteorological literature formulae (19) and (18) are written for $\gamma_d \equiv p_v/p_d$ instead of for $\gamma \equiv p_v/p$ (3). At $\gamma \ll 1$ we have $\gamma \approx \gamma_d$, but at $p_d \to 0$ we have $\gamma_d \to \infty$, which leads to a physically meaningless expression $\partial\gamma_d/\partial z \to \infty$ if one replaces γ by γ_d in the continuity equation (9) and Eqs. (20) and (2).

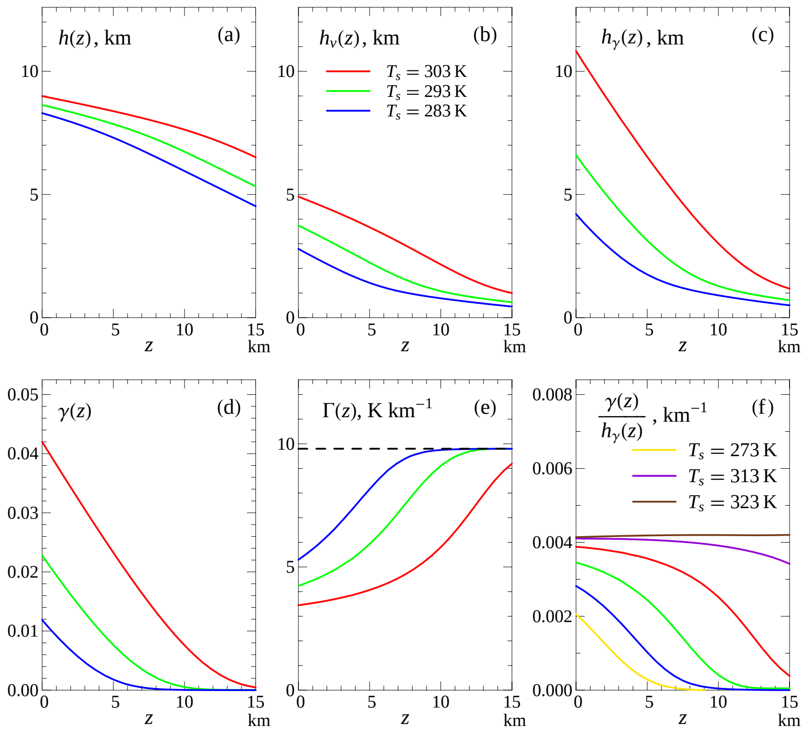

Figure 2: Dependence on height z of the major distribution functions of moist air for several different values of surface temperature T_s satisfying the system of equations (19)-(20).

(a): $h(z)^{-1} = -\dfrac{1}{p}\dfrac{\partial p}{\partial z}$ is the scale height (17) of the moist air pressure;

(b): $h_v(z)^{-1} = -\dfrac{1}{p_v}\dfrac{\partial p_v}{\partial z}$ is the scale height (16) of the partial pressure of saturated water vapor;

(c): $h_\gamma \equiv -\dfrac{1}{\gamma}\dfrac{\partial \gamma}{\partial z}$ is the scale height of water vapor condensation (20); p and p_v are pressures of moist air and saturated water vapor, respectively.

(d): $\gamma(z) \equiv p_v(z)/p(z)$ is the mixing ratio of saturated water vapor;

(e): $\Gamma(z) = -\partial T/\partial z$ is the moist adiabatic lapse rate of air temperature, the dashed line indicates the dry adiabatic lapse rate $\Gamma_d = 9.8$ K km^{-1};

(f): $\gamma(z)/h_\gamma(z) = -\partial\gamma/\partial z$ is the relative intensity of water vapor condensation.

gas; unlike oxygen, gaseous nitrogen is chemically inert and cannot lead to excessive oxidation or fires in the biosphere. The major role of atmospheric nitrogen consists in making the value of γ small at a fixed temperature. The condensation-induced circulation makes the biotic pump of atmospheric moisture (Makarieva and Gorshkov, 2007) and a hydrological cycle on land possible, thus ensuring that land is habitable for life. This suggests that the existing concentration of atmospheric nitrogen should have been formed by the time while life started colonizing the land and the continental forest cover developed.

Note that if $\gamma \to 1$ due to increasing temperature (and not due to the removal of the dry air component), then the exponential growth of $p_v = \gamma p$ in accordance with the Clausius-Clapeyron equation (16) makes the evaporative-condensational force grow exponentially as well, such that the condensation-induced atmospheric circulation is ensured at any small share of the dry component in the total air pressure, see (12), (14) and Fig. 2f.

For a given value of $\gamma(0) \equiv \gamma_s \ll 1$ at the surface, $\gamma(z)$ declines rapidly with increasing z, Fig. 2d. Accordingly, the adiabatic temperature lapse rate $\Gamma(z)$ (19) rises from its minimal value at the surface approaching the dry adiabatic lapse rate Γ_d at large heights, Fig. 2e. Any circulation pattern includes areas where the air masses rise (in the region where evaporation and condensation are more intense) and areas where the air masses descend (in the region where evaporation and condensation are less intense). In the region of descent the air masses warm adiabatically while descending, so condensation cannot occur. Consequently, neglecting horizontal mixing, the adiabatic temperature lapse rate in the region of descent should be equal to the dry adiabatic lapse rate. Air temperature T and air pressure p related to temperature by the equation of state (1) decrease with height more slowly in the region of ascent than they do in the region of descent. Pressure difference Δp between the regions of ascent and descent has a negative value at the surface, then decreases by absolute magnitude with growing height, approaches zero and changes its sign at a certain height $z_c \sim h_\gamma$, where horizontal velocity u changes its direction (Makarieva et al., 2010b).

The equation of state (1) can be written in the following form

$$p = \rho g h, \quad \rho = NM, \quad h = RT/(Mg). \tag{21}$$

This relationship for p does not in reality depend on either M or g, which actually cancel in (21). But written in this form, presure has a simple physical meaning of potential energy in the gravity field, where h represents the height of atmospheric column, ρg represents weight of a unit air volume in the air column, while p represents the total weight of the air column of a unit area. Relationshp (21) holds for any height z, with ρ and T depending on z. At the Earth's surface $z = 0$; surface values of all variables are denoted by low index s. The difference in surface weights between two hypothetical static air columns – in one of which saturated water vapor has condensed at all heights, while in the other no condensation took place – is equal to the difference of the right-hand parts of the first equality in (21) written for the two columns:

$$p_{vs}gh_{vs} - p_{vs}gh_s = p_{vs}g(h_{vs} - h_s) < 0. \qquad (22)$$

Here surface pressure p_{vs} of water vapor is set to be saturated, so its value is determined by the surface temperature and is the same in the two columns considered. It follows from (22) that the first column, where condensation took place, cannot remain in static equilibrium, as surface air pressure appears to be larger than the column weight. The surplus of pressure at the surface causes air to rise and all water vapor to ultimately condense as the air rises and cools. This illustrates the physical role of water vapor contained in the air column as a store of potential energy available for atmospheric circulation.

The global mean store of potential energy per unit atmospheric water vapor mass is of the order of $\overline{p_{vs}/\rho_{vs}} \approx R\overline{T_s}/M_v \sim 1.3 \times 10^5$ J (kg H_2O)$^{-1}$ at the global mean surface temperature $\overline{T_s} \approx$ 288 K. The global mean precipitation rate is $\Pi \sim 10^3$ kg H_2O m^{-2} year^{-1} (L'vovitch, 1979). Thus, the global rate of potential energy release associated with water vapor condensation is of the order of $\Pi R\overline{T_s}/M_v \sim 4$ W m^{-2}. This potential energy flux is sufficient to drive the general atmospheric circulation of Earth. The power of the latter has been estimated at around $\sim 1\%$ of the global power of 2.4×10^2 W m^{-2} of the absorbed solar radiation (Lorenz, 1967).

4 Large-scale and compact circulations

In a stationary circulation condensation of water vapor in the region of adiabatic ascent of air masses must be compensated by an inflow of water vapor from the hydrosphere to the atmosphere via evaporation. This follows from the general continuity equation written in the form of (8) or (9). If γ does not change in the horizontal direction, the change of γ in the vertical direction that is due to condensation, also turns to zero.

In a large-scale circulation where pressure gradient forces are compensated by friction forces such that horizontal velocity u does not change, the maintenance of a constant value of water vapor partial pressure in the horizontally moving air masses, $\partial p_v/\partial x = 0$, is achieved via continuous local evaporation. Stipulation $\partial p_v/\partial x = 0$ describes circulation that is isothermal in the horizontal direction. Evaporation flux should be of the same order of magnitude as the condensation flux. The latter exceeds the evaporation flux by no more than approximately twofold – at the expense of horizontal import of water vapor evaporated within the region of descent where condensation does not take place. The relative value of evaporation dictates the direction of air circulation: the air flow in the lower atmosphere is directed towards the region of higher evaporation, which entails more intense condensation.

Namely this pattern determines how the biotic pump of atmospheric moisture functions. Evaporation from the forest canopy of natural forests can exceed evaporation from the open oceanic surface by over twofold. This leads to the appearance of atmospheric flow bringing moisture from the ocean to land to compensate for the continental moisture loss due to gravitational runoff (Gorshkov and Makarieva, 2006; Makarieva and Gorshkov, 2006, 2007; Makarieva et al., 2009; Makarieva and Gorshkov, 2009a), Fig. 3. Intense condensation over forest canopy creates a region of low pressure of continental scale that sustains the ocean-to-land air flow. The regular and persistent atmospheric moisture delivery of the biotic pump becomes possible due to the fact that natural forests control the processes of evaporation (via transpiration and intercept) and condensation (via production and emission of biogenic condensation nuclei). This regular moisture transport does not allow spatial and temporal fluctuations of condensation to develop thus preventing

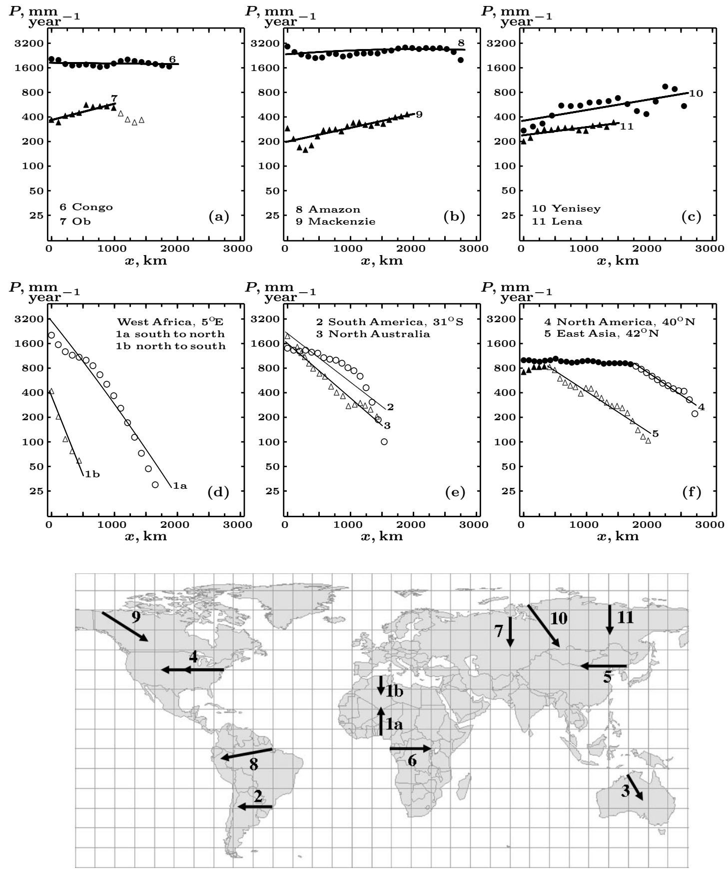

Figure 3: Biotic pump of atmospheric moisture in river basins covered by natural forests (black symbols) as compared to disturbed and unforested regions (open symbols). Shown is the mean annual precipitation on land associated with large-scale regional atmospheric circulation patterns as dependent on distance from the ocean. Arrows on the map indicate which regions were considered.
(a), (b), (c): data for the world's largest river basin covered by natural forests. In the basins of world's largest rivers – Amazon and Congo – precipitation is twice higher than over the ocean and does not decrease with growing distance from the ocean. Precipitation in the basins of the northern rivers that flow from the south grow with distance from the ocean proportionally to the increase in solar radiation (Makarieva et al., 2009). In the Ob basin precipitation declines in the deforested region near the place where Ob is joined by Irtysh.
(d), (e), (f): precipitation over non-forested areas decline exponentially with distance from the ocean. In panel (f) temperate forests of the North America maintain nearly constant precipitation until the deforested region is reached (line 4).

the formation of hurricanes and tornadoes over the natural forest canopy spread over millions of square kilometers. All these biotic pump features disappear in deforested regions, that, via the transitional monsoon-type regime of intermittent drafts and floods, ultimately turn to deserts that are deprived of precipitation altogether, Fig. 3.

In a compact circulation, friction forces are negligibly small compared to pressure gradient forces. This makes it possible for the air masses to accelerate and achieve catactrophic velocities observed within hurricanes and tornadoes. In this case condensation rate can exceed the local evaporation rate as supported by solar radiation by several orders of magnitude. The circulation pattern can remain stationary if only, after the local store of water vapor is depleted, the pressure field moves along to the adjacent area where water vapor is still abundant. Having depleted the water vapor in the new area, the pressure field moves to the next area and consumes water vapor there, and so on (Gorshkov, 1990, 1995; Gorshkov et al., 2000). Thus, in this case the import of water vapor to the circulation area occurs dynamically via movement of the pressure field, while the consumption of water vapor occurs thermodynamically in the course of adiabatic ascent of air masses.

Hurricane energetics is determined by conservation of the sum of kinetic energy of the radial, tangential and vertical air movement and potential energy of condensing water vapor on each streamline, as described by the Bernoulli integral. (While Euler's equations are non-linear, Bernoulli's integrals that represent the sum of kinetic and potential energy, are linear. Therefore, any bunch of streamlines (sum of particular streamlines) is also a Bernoulli integral.) As friction is negligible, angular momentum is conserved along each streamline. The apparent non-conservation of angular momentum within the hurricane is caused by a superposition of different streamlines that correspond to different boundary conditions (see Section 8).

Re-distribution of energy between the radial, tangential and vertical velocity components of the air flow results in the appearance of an eye of hurricane or tornado near the condensation center. Due to angular momentum conservation, kinetic energy of the tangential flow grows more rapidly towards the center than does kinetic energy corresponding to the radial and vertical velocity. Therefore, all energy is ultimately converted to the energy of the tangential air flow, while the radial and vertical flow components cease to exist (their energy becomes zero). As far as condensation rate and the associated potential energy are proportional to vertical velocity, condensational potential energy turns to zero as well. All the energy store of the Bernoulli integral is now concentrated in the kinetic energy of the tangential air flow. The rotating air flow cannot move further towards the center because of the zero radial velocity, so a spot of calm weather is formed in the center. In the absence of considerable friction, the kinetic energy of the tangential air flow can persist for a long time endangering any object that can become a source of frictional dissipation. We summarize that the properties of hurricanes and tornadoes follow from (1) the properties of the potential energy associated with condensation, (2) conservation of angular momentum, (3) conservation of air flow (continuity equation) with a high precision, and cannot be understood without taking the corresponding physical processes into account.

5 Condensational pressure gradient in a radially symmetrical circulation

Let us now consider the structure of hurricanes and tornadoes in greater quantitative detail. Such compact circulations have a conspicuous condensation center and can be considered in the cylindrical system. The continuity equation in the cylindrical coordinates under the assumption of radial symmetry (no dependence on angle φ) have the following form, cf. (4)-(6):

$$\frac{1}{r}\frac{\partial Nur}{\partial r} + \frac{\partial Nw}{\partial z} = w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right). \tag{23}$$

$$\frac{1}{r}\frac{\partial N_d ur}{\partial r} + \frac{\partial N_d w}{\partial z} = 0, \tag{24}$$

$$\frac{1}{r}\frac{\partial N_v ur}{\partial r} + \frac{\partial N_v w}{\partial z} = w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right). \tag{25}$$

Here u and w are the radial and vertical velocities of the air flow. Due to radial symmetry, tangential velocity v does not enter (23)-(25). Expanding the derivatives in Eq. (25) and multiplying both parts of the equation by N/N_v we obtain

$$u\frac{N}{N_v}\frac{\partial N_v}{\partial r} + w\frac{\partial N}{\partial z} + N\left(\frac{1}{r}\frac{\partial ur}{\partial r} + \frac{\partial w}{\partial z}\right) = 0. \tag{26}$$

Now expanding the derivatives in the left hand part of Eq. (23) and using Eq. (26) we obtain the following continuity equation where condensation is taken into account:

$$u\left(\frac{\partial N}{\partial r} - \frac{N}{N_v}\frac{\partial N_v}{\partial r}\right) = w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right). \tag{27}$$

Using the equation of state (1) we finally have

$$u\left(\frac{\partial p}{\partial r} - \frac{1}{\gamma}\frac{\partial p_v}{\partial r}\right) = wp\gamma\left(\frac{1}{p_v}\frac{\partial p_v}{\partial z} - \frac{1}{p}\frac{\partial p}{\partial z}\right) \tag{28}$$

or

$$u\frac{\partial \ln\gamma}{\partial r} = -w\frac{\partial \gamma}{\partial z}. \tag{29}$$

When the condensation area is horizontally isothermal, we have $\partial p_v/\partial r = 0$, so Eq. (28) takes the form similar to (11)-(12):

$$-\frac{\partial p}{\partial r} = \frac{w}{u}\frac{\Delta p}{h_\gamma},\ \ \Delta p \equiv p\gamma = p_v,\ \ h_\gamma^{-1} \equiv h_v^{-1} - h^{-1}, \tag{30}$$

$$h_v^{-1} \equiv -\frac{1}{p_v}\frac{\partial p_v}{\partial z},\ \ h^{-1} \equiv -\frac{1}{p}\frac{\partial p}{\partial z},\ \ \frac{\partial p_v}{\partial r} = 0, \tag{31}$$

where scale height h_v and h are determined from the conditions of adiabatic ascent and hydrostatic equilibrium for moist air including the remaining non-condensed water vapor, see Section 3. As in (11)-(12), h_γ has the meaning of the scale height where most part of water vapor condenses. All variables entering (30) and (31) depend on r and z.

To find the dependence of pressure gradient (30) on r and z let us use the continuity equation in the integral form. Air flow converging towards the condensation center penetrates via vertical round wall of radius r, circumference $2\pi r$ and height h_γ and leaves the condensation area via horizontal disk of area πr^2:

$$2\pi r h_\gamma u N = 2\pi\int_0^r N_d w r dr,\ \ \text{or}\ \ \frac{1}{r}\frac{\partial N u r}{\partial r} = N_d w. \tag{32}$$

Here we took into account that all water vapor that entered the circumfurence has condensed and it is only the dry air component that leaves the condensation area. Recalling the smallness of $\gamma \ll 1$ and considering that the relative pressure difference between the hurricane center and the outskirts does not exceed 10%, one can neglect the corresponding changes of $N \approx N_d$ in (32). Then we have:

$$\frac{w}{h_\gamma} = \frac{1}{r}\frac{\partial ur}{\partial r} = \frac{u}{r} + \frac{\partial u}{\partial r}. \qquad (33)$$

Putting (33) into (30) we obtain

$$-\frac{\partial p}{\partial r} = \frac{\Delta p}{ur}\frac{\partial ur}{\partial r} = \Delta p \frac{\partial \ln ur}{\partial r}, \quad \Delta p \equiv p\gamma \equiv \frac{1}{2}\rho u_c^2, \qquad (34)$$

where p and γ are the corresponding values at the surface or, more accurately, at the height of boundary layer where relative humidity reaches unity and condensation commences; u_c is the condensational velocity, which is the velocity scale that determines the magnitude of kinetic energy to which potential energy Δp can be converted.

6 Profiles of pressure and velocities in hurricanes and tornadoes

Thus, potential energy $P(r)$ that is responsible for the formation of hurricanes and tornadoes, depends on radial velocity $u(r)$ (the latter itself appears due to conversion of potential energy $P(r)$ to kinetic energy) and is equal to

$$P(r) = p(r) = \Delta p \ln ur + p(r_p). \qquad (35)$$

The Bernoulli integral of Euler equation for a streamline has the form

$$\Phi(r) \equiv \frac{1}{2}\rho_s(u^2 + v^2 + w^2) + \Delta p \ln ur = \Phi(r_p), \qquad (36)$$

where r_p is a fixed radius, v is tangential velocity that is perpendicular to radius r. As a boundary condition to fix r_p it is natural to set $u(r_p) \sim 5$ m s^{-1} to be equal to the average wind velocity outside the area of the considered compact circulation. Conservation of angular momentum constrains the dependence of v on r. Using the continuity equation (33) that relates w and u to each other, we then have

$$vr = v_p r_p, \;\; v = v_p \frac{r_p}{r}, \;\; \frac{w}{h_\gamma} = \frac{1}{r}\frac{\partial ur}{\partial r}. \tag{37}$$

We now go over to the following dimensionless variables:

$$x \equiv \frac{r}{r_p}, \;\; \Delta p \equiv \frac{1}{2}\rho u_c^2, \;\; u_c \equiv \frac{2\Delta p}{\rho} = \gamma\frac{2p}{\rho}, \tag{38}$$

$$u \to \frac{u}{u_c}, \;\; v \to \frac{v}{u_c}, \;\; w \to \frac{w}{u_c}, \;\; \frac{\partial p}{\partial r} \to \frac{1}{\Delta p}\frac{\partial p}{\partial x}, \;\; p \to \frac{p}{\Delta p}, \tag{39}$$

$$\frac{v_p}{u_c} = \frac{A_p}{A_c} \equiv a, \;\; v_p \equiv v(r_p), \;\;\; A_p \equiv v_p r_p, \;\;\; A_c \equiv u_c r_p. \tag{40}$$

Here u_c is the condensational velocity scale (34), A_c is the condensational angular momentum, a represents the dimensionless angular momentum $A = A_p$ (per unit mass density ρ) that is conserved.

Leaving the pressure and velocity notations unchanged for the dimensionless variables (which is equivalent to choosing the units u_c and Δp of velocity and pressure measurements equal to unity) we obtain for (36):

$$\Phi(x) \equiv u^2 + v^2 + w^2 + \ln ux = \Phi(1), \tag{41}$$

$$v = \frac{a}{x}, \;\; w = \beta\frac{1}{x}\frac{\partial ux}{\partial x}, \;\; \beta \equiv \frac{h_\gamma}{r_p}, \;\; p(x) = \ln ux + p(1). \tag{42}$$

Putting (42) into (41) we finally obtain the following equation on radial velocity u:

$$\Phi(x) \equiv u^2 + \frac{a^2 + \beta^2 u^2}{x^2} + 2\beta^2 \frac{u}{x}\frac{\partial u}{\partial x} + \beta^2 \left(\frac{\partial u}{\partial x}\right)^2 + \ln ux = \Phi(1). \tag{43}$$

Due to the Earth's rotation, in the inertial frame of reference the air volumes rotate with an angular velocity $\omega = \Omega \sin\vartheta$, where ϑ is the latitude angle where circulation takes place, $\Omega = 2\pi/\tau$, $\tau = 24$ h. Far from the condensation area due to friction effects the rotation is similar to solid body rotation. There is no radial convergence of the air flows towards the condensation center. Angular momentum is proportional to ωr^2 and declines proportionally to r^2 with decreasing r. Radial velocity and air convergence towards the condensation center arise at $r = r_p$, where the pressure gradient forces associated with the on-going condensation significantly exceed the friction forces. Starting from $r < r_p$ angular momentum in the inertial frame proportional to $\omega r_p^2 = v_p r_p = vr$, $v_p = \omega r_p$, is conserved. Tangential velocity increases towards the center in accordance to (37) as a consequence of the radial symmetry of the condensation-induced pressure gradient forces and the smallness of friction forces that can be put approximately equal to zero at $r < r_p$.

In the system of observations, due to friction between the air and the surface, the air is motionless at $r > r_p$ and the angular momentum is equal to zero. At $r < r_p$ there appears a non-zero radial velocity and the air masses start to converge towards the center. At this moment the Coriolis force that is perpendicular to radial velocity vector and to the vector of angular velocity is non-central; it starts to curl the converging air masses, which leads to the increment of tangential velocity and angular momentum. This occurs until tangential velocity becomes considerably larger than radial velocity. Then the Coriolis force becomes a central force, and the angular momentum is further conserved.

Tangential velocities in the inertial frame of reference v ($v = v_p r_p / r = \omega r_p^2 / r$) and in the system of observations v_{ob} are related as follows:

$$v_{ob} = v - \omega r = \omega \left(\frac{r_p^2}{r} - r \right) = v_p \left(\frac{1}{x} - x \right) = v_p \frac{1 - x^2}{x}, \;\; x \equiv \frac{r}{r_p}. \tag{44}$$

Equation (44) describes the fact that in the system of observations from the value of tangential velocity in the inertial frame, where angular momentum is conserved, one substracts tangential velocity equal to the velocity of a solid body rotation ωr. Angular momentum per unit mass density ρ in the system of observations is proportional to $v_{ob} r$:

$$A_{ob} = v_{ob} r = \omega r_p^2 \left(1 - \frac{r^2}{r_p^2} \right) = A(1 - x^2), \tag{45}$$

where $A = \omega r_p^2 = v_p r_p$ is the angular momentum that is conserved in the inertial system. Change of angular momentum in the system of observations as the air moves radially towards the center is determined by Coriolis force:

$$\frac{d\boldsymbol{A}_{ob}}{dt} = -2[\boldsymbol{r}[\boldsymbol{\omega}\,\boldsymbol{V}(\boldsymbol{r})]] = -2\boldsymbol{\omega}ur, \quad \boldsymbol{V} = \boldsymbol{u} + \boldsymbol{v} + \boldsymbol{r}, \ \ (\boldsymbol{V}\boldsymbol{r}) = ur,$$
$$A_{ob}(r) = \int_{r_p}^{r} \frac{dA}{dt}dt = \int_{r_p}^{r} \frac{dA}{dt}\frac{dt}{dr}dr = -\int_{r_p}^{r} 2\omega ur\frac{1}{u}dr =$$
$$= \omega r_p^2\left(1 - \frac{r^2}{r_p^2}\right) = A(1 - x^2),$$
$$u = \frac{dr}{dt}, \ \ A_{ob}(r_p) = 0, \tag{46}$$

which coincides with Eq. (44). Therefore, at $x \ll 1$ ($r \ll r_p$) the angular momenta in the system of observations and the inertial system are conserved and coincide. Due to this fact it is convenient to consider hurricanes and tornadoes in the inertial system where the Coriolis force is absent. Thus, the conserved dimensionless angular momentum a is defined as follows, see (40):

$$a \equiv \frac{r_p\omega}{u_c}, \ \ a_{ob} = a(1 - x^2) \approx a, \ \ x \ll 1. \tag{47}$$

An important peculiarity of a compact circulation like hurricanes and tornadoes is the fact that in order for the circulation to remain stationary (independent of time), it is necessary that the entire circulation pattern moves as a whole with velocity U along the Earth's surface. Within each streamline the air becomes deprived of water vapor after it ascends in the condensation area. When all the air in the condensation area ascends to height h_γ, the circulation pattern has to move over a horizontal distance of the order of the circulation size r_p. Consequently, the horizontal velocity of the system movement is $U \sim \overline{w}r_p/h_\gamma$, where $\overline{w}$ is the mean vertical velocity of the ascending air within the circulation pattern.

In hurricanes the vertical velocity is small, so $\beta \ll 1$, see (42), and can be put equal to zero in (43). Then Eq. (43) assumes a simple form:

$$u^2 + \frac{a^2}{x^2} + \ln\left(\frac{u}{u_1}x\right) = a^2 + u_1^2, \ \ p(x) = \ln\left(\frac{u}{u_1}x\right), \ \ u_1 \equiv u(1). \tag{48}$$

Vertical velocity w is related to radial velocity u by Eq. (42). Point of maximum $u = u_m$ that is obtained from $\partial u/\partial x = 0$ corresponds to $x = x_m = \sqrt{2}a$. At smaller $x < x_m$ the value of u^2 rapidly declines with decreasing x. Contribution of the logarithmic term $-\ln(u/u_1)$ vanishes as u decreases down to $u_1 < u_m$. Further increment of $-\ln(u/u_1)$ at $u < u_1$ does not have a physical meaning. Indeed, since the water vapor is depleted in the end of the streamline at some small x, the condensational potential turns to zero. We thus assume that $x = x_0$, where radial velocity decreases from its maximum value to $u = u_0 = u_1$, is the point where the condensational pressure gradient ceases to act (see Section 9). At this point the radial and vertical velocities as determined by condensation within the considered area are effectively equal to zero. All condensational energy is concentrated in the kinetic energy that corresponds to the tangential velocity of the air flow.

7 Rotation of the eye

According to Eq. (48), the value of x_0 at which $u = u_0 = u_1$ is a root of the following equation, Fig. 4:

$$-\ln x_0 = \frac{a^2}{x_0^2}(1 - x_0^2) \approx \frac{a^2}{x_0^2}, \;\; x_0^2 \ll 1; \;\; x_0 = x_0(a). \qquad (49)$$

Point x_0 determines the radius of the hurricane eye for the case when the air in the eye were motionless with a zero kinetic energy of rotation. However, in this case at $x = x_0$ the tangential velocity reaches its maximum value $v_0 = a/x_0 = \omega_0 x_0$. Therefore, the air in the eye cannot be motionless being in contact with the rotating windwall at $x = x_0$. In the absence of friction losses between the eye and the windwall, the eye should rotate as well with a constant angular velocity ω_0 determined by the tangential velocity of the windwall at $x = x_0$. Energy for the rotation of the eye can be only borrowed from the kinetic energy of the rotating windwall, because at $x = x_0$ the radial and vertical velocities, as well as the potential energy of condensation, are close to zero. The energy for eye rotation is therefore a certain share of the full potential energy of water vapor condensation that was converted to the kinetic energy of the windwall[9,10].

Transfer of kinetic energy to the eye reduces the maximum tangential velocity of the hurricane that is observed within the windwall from v_0 to $v_e = a/x_e$. The eye radius expands from x_0 to $x_e > x_0$. The air in the eye rotates with a constant angular velocity $\omega_e = v_e/x_e < \omega_0$ and tangential velocity $v = \omega_e x$, $x < x_e$. For the definitiveness sake, below we continue to call x_0 as the radius of the (motionless) eye while referring to x_e as to the radius of the windwall (of the rotating eye). According to the energy conservation law, the kinetic energy of eye rotation should be equal to the kinetic energy of rotation of the windwall segment $x_0 \leq x \leq x_e$ with tangential velocity $v = a/x$ which would take place if the eye remained motionless, see Fig. 5a,b. Neglecting the air density change in the eye as compared to the windwall (see footnote 5) and cancelling the common multiplier 2π, this stipulation can be written in

[9] The pressure gradient force is perpendicular to isobars. Coriolis force is perpendicular to the velocity vector and is proportional in magnitude to velocity. If the two forces act in the opposite directions and compensate each other, one can determine the constant value of velocity corresponding to movement along the isobars. In the inertial frame of reference for the case of radially symmetrical closed isobars this corresponds to the equality between the centrifugal force that depends on constant tangential velocity v and the pressure gradient force: $v^2/r + \partial p/\partial r = 0$ (the so-called cyclostrophic balance). If there is no friction, the angular momentum is conserved, and the rotational movement with zero radial velocity and constant tangential velocity can continue infinitely, similarly to satellite rotation on a terrestrial orbit. Friction decreases tangential velocity and, hence, the centrifugal force; this disturbes the balance of forces and causes some radial movement towards the center of the isobars. In the existing theories of hurricane formation the pressure gradient force is approximated from the condition of approximate cyclostrophic balance using the dependence of tangential velocity on radius and the fact that the observed radial and vertical velocities are small compared to the former. Friction is considered as the major cause of the convergence of air masses towards the hurricane center (e.g., Smith et al., 2008). The cause-and-effect link between the radial convergence and the existence of the radial pressure gradient is neglected. However, nature works the other way round: it is namely the radial convergence of moist air with a non-zero radial velocity u and the physically inseparable ascent of air masses with vertical velocity w, see (33), that determine the pressure fall from the outer environment towards the hurricane center, see (34). If there is no radial convergence and the radial velocity is zero, then there is no ascent of moist air, no condensation of water vapor, no condensation-induced pressure gradient, no close isobars and no approximate cyclostrophic balance – in short, no circulation and, consequently, no friction. This is the major difference of the condensational potential from the velocity-independent gravitational potential that keeps the rotating satellite on its orbit. The condensational potential energy as the air converges towards the center is mainly converted to the kinetic energy of rotation determined by tangential velocity v. This results in the development of huge velocities namely due to the absence of friction. Friction can only lead to dissipation of this energy impeding the air acceleration and preventing the development of maximum possible velocities.

[10] The appearance of eye rotation due to the contact of the eye with the rotating windwall represents a re-distribution of the ordered kinetic energy. It cannot not be considered as turbulent friction losses (i.e., energy losses on the formation of the chaotic turbulent air flows).

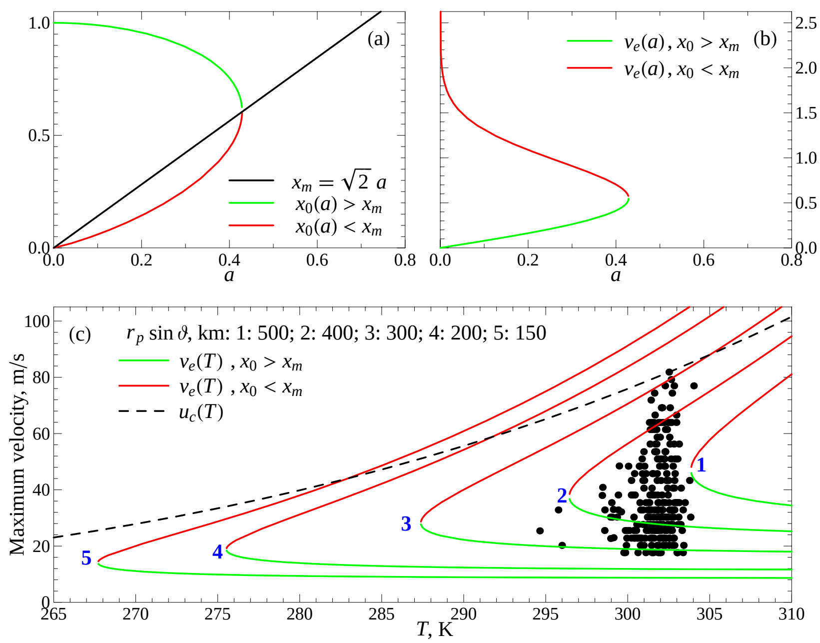

Figure 4: The regime of hurricane existence corresponds to the two solution branches of equation (49). The green curves in all panels correspond to the branch where the eye radius $x_0(a)$ grows up to $x_0 = \sqrt{2}a = 0.6$ as a increases up to $a \approx 0.43$. The red curves in all panels correspond to the branch where the eye radius grows from $x_0 = 0.6$ to $x_0 = 1$ with a diminishing from 0.43 to zero.
(a): Two branches of the solution of the approximate equation (49).
(b): Maximum dimensionless tangential velocity $v_e(a) = a/x_e = a/(1.28x_0)$ (51). The branch with lower velocity corresponds to widening of the eye accompanied by the decrease of condensational energy.
(c): Absolute values of maximum tangential velocities $v_e(a) = u_c a/x_e$ depending on temperature for five different values of parameter $r_p \sin\vartheta$. For example, $r_p \sin\vartheta = 300$ km corresponds to a streamline with the outer radius $r_p = 600$ km formed at latitude $\vartheta = 30^o$. The value of u_c grows exponentially with increasing temperature as $u_c = [2\gamma(T)RT/M]^{1/2}$, see (38), $\gamma(T) \equiv p_v(T)/p$, $p_v(T)$ conforms to the Clausius-Clapeyron equation $dp_v/p_v = (L/RT)dT/T$, see (16), $\gamma = 0.042$ at $T = 303$ K.
Black dots represent the data from Fig. 3 of Michaels et al. (2006) that correspond to maximum wind velocities observed in 270 tropical cyclones.

the following form, see Fig. 5a:

$$\omega_e^2 \int_0^{x_e} x^2 x dx = a^2 \int_{x_0}^{x_e} \frac{x dx}{x^2}, \ \ \omega_e^2 = \frac{a^2}{x_e^4}. \tag{50}$$

Performing the integration and cancelling the common multiplier a^2 we obtain

$$\ln \frac{x_e}{x_0} = \frac{1}{4}, \ \frac{x_e}{x_0} = e^{0.25} = 1.28; \ x_e(a) = 1.28 x_0(a), \ v_e(a) = \frac{a}{1.28 x_0(a)}. \tag{51}$$

Tangential velocity v grows with decreasing x at $x > x_e$ and declines with decreasing x at $x < x_e$ as follows, see (47):

$$v = \begin{cases} \dfrac{a}{x}, & x \geq x_e, \ vx = a = \dfrac{\omega r_p}{u_c} = \text{const.}, \omega = \Omega \sin \vartheta, \\ \\ \dfrac{ax}{x_e^2}, & x \leq x_e, \ vx = \dfrac{ax^2}{x_e^2} \neq \text{const.} \end{cases} \tag{52}$$

The dependence of the maximum value of tangential velocity $v_e = v(x_e)$ on a is shown in Fig. 4b.

Stationary existence of the rotating windwall and the rotating eye modifies pressure profile $p(x)$ at $x \leq x_e$ in such a manner that the pressure gradient force and the centrifugal force within the eye coincide. In order that an additonal pressure drop could form within the eye where no condensation takes place, some part of the air must be exported away from the eye outside the hurricane area ($x > 1$) during the eye formation in such a manner that the parts of the pressure profile inside and outside the rotating eye joined smoothly and featured no discontinuity at the windwall $x = x_e$. This condition can be written as (Makarieva and Gorshkov, 2009b)

$$\frac{\partial p}{\partial x} = 2\frac{v^2}{x} = 2\omega_e^2 x, \ \ p(x) = \omega_e^2 x^2 + p(0), \ \ \omega_e = \frac{a}{x_e^2}, \ \ x \leq x_e; \tag{53}$$

$$p(x) = \ln \left(\frac{u}{u_1} x \right), \ \ p(1) = 0, \ \ x \geq x_e. \tag{54}$$

(Multiplier "2" in (53) arises from the equation $\partial p / \partial r = v^2 / r$ as one goes to the dimensionless variables (39): dividing both parts of the equality by $\Delta p = \rho_s u_c^2 / 2$ and changing to the dimensionless velocity

v/u_c.) In Eq. (54) the condensational potential is set equal to zero outside the condensation area $x \geq 1$ ($r \geq r_p$). Thus, $p(x)$ describes how air pressure declines from its normal value it has outside the hurricane. Accordingly, $-p(0) = \delta p$ is equal to the total (dimensionless) pressure fall within the hurricane including the pressure fall associated with the eye rotation. In ordinary units, the pressure fall is equal to $\delta p \Delta p = \gamma p \delta p$. Combining (53) and (54) we have, see Fig. 5c,d:

$$p(x) = \begin{cases} \ln\left(\dfrac{u}{u_1}x\right), & x \geq x_e \\ \\ a^2\dfrac{(x^2 - x_e^2)}{x_e^4} + \ln\left(\dfrac{u}{u_1}x\right), & x \leq x_e \end{cases}, \tag{55}$$

$$\delta p = \frac{a^2}{x_e^2} - \ln\left(\frac{u_e}{u_1}x_e\right). \tag{56}$$

The first and second terms in (56) represent the pressure fall within the eye and outside the eye, respectively. At $a = 0.12$ ($r_p = 400$ km, $\vartheta = 20^{\rm o}$, $u_c = 83$ m s^{-1} at $\gamma \approx 0.04$) we have $x_0 \approx 0.12$, see Fig. 4a; $x_e \approx 0.15$, see (51); $u_e/u_1 \sim 2$, see Fig. 5b. Thus, from Eq. (56) we obtain $\delta p \approx 3$, Fig. 5d. For the absolute pressure fall we have $\delta p \Delta p/p \sim 3\gamma \sim 0.13$ at $\gamma \approx 0.04$, which means that the air pressure in the eye center is at maximum 13% lower than the air pressure outside the hurricane.

Note that the two branches of pressure $p(x)$ (55) inside and outside the eye join smoothly at $x = x_e$, while the derivative $\partial p/\partial x$ features a minor discontinuity at this point. This discontinuity is practically unnoticeable in Fig. 5d. This discontinuity has a clear physical meaning, but it is difficult to observe it at small a. If using the smallness of a, x_e and x_m one chooses $x_e = x_m$ at the point where radial velocity has its maximum, then the derivatives $\partial p/\partial x$ also coincide in this point, see (49). This is a consequence of Eq. (48) and of the fact that $\partial u/\partial x = 0$ at $x = x_m$ (Makarieva and Gorshkov, 2009b).

Equation (49) represents an equation on the value of x_0 as a function of a (51), $x_0 = x_0(a)$, Fig. 4a. The value $x_0 = x_m = 0.6$ is reached at $a = 0.43$, where $x_0^2 = 0.36$ and the approximation (49) still holds. Further widening of the eye radius x_0 involves the

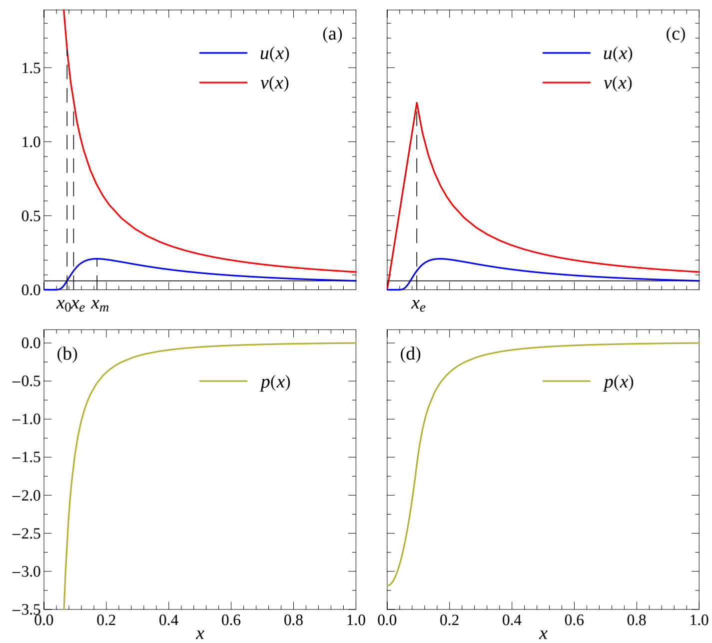

Figure 5: Dimensionless radial u (48) and tangential v (52) velocities and pressure fall $p(x)$ (55) as dependent on radius to the hurricane center x.
(a), (b): velocities and pressure assuming the eye is motionless;
(c), (d): velocities and pressure with an account of eye rotation;
x_0 is eye radius (the eye is motionless);
x_e is windwall radius (the eye is rotating);
x_m is the point where radial velocity reaches its maximum;
$a = r_p\omega/u_c$ is the dimensionless angular momentum that is conserved on the streamline outside the eye; r_p is the outer radius of the hurricane streamline; $u_c = (2\Delta p/\rho)^{1/2}$ is the condensational velocity scale, see (38) and notes to Fig. 4c; $\Delta p = \gamma p$ is pressure drop due to condensation, $\gamma \equiv p_v/p$ is the relative share of water vapor, p is air pressure outside the hurricane.
The curves correspond to the following parameters: $r_p = 400$ km, $\omega = \Omega \sin\vartheta = 2.5\times 10^{-5}$ s^{-1}, $\vartheta = 20^{\rm o}$, $\gamma = 0.042$, $T = 30^{\rm o}$ C, $\rho = 1.22$ kg m^{-3}, $p = 10^5$ J m^{-3}, $a = 0.12$, $u_c = 83$ m s^{-1}. The black horizontal line in panels (a) and (c) denotes $u_1 = 0.06$.

second branch of the solution of Eq. (49) shown in Fig. 4a. For this branch the growth of x_0 to $x_0 = 1$ corresponds to decreasing a. On this branch potential energy arising from condensation that occurs in the area $x_0 \leq x \leq 1$ and disappears at $x_0 = 1$, $a = 0$, should have been spent on rotation of a very wide eye at low tangential velocities, i.e., the hurricane cannot form.

Transition from this branch with $x_0 > x_m$ to the branch $x_0 < x_m$ corresponds to formation of a hurricane. In Fig. 4c the areas of possible maximum tangential velocities are shown for several values of $r_p \sin\vartheta$ as dependent on oceanic surface temperature T_s that dictates the value of condensational velocity u_c (38). We note two peculiarities: (1) temperature that corresponds to the transition between the two regimes decreases with diminishing $r_p \sin\vartheta$ and (2) for any given temperature, the difference between velocities of the two branches grows with decreasing $r_p \sin\vartheta$. In the equatorial zone where $\sin\vartheta \to 0$, at observed surface temperatures the difference between velocities on the two branches tends to infinity. Hurricanes cannot form.

As one can see from Fig. 4c, for any given temperature, the distance between the branches increases (while the probability of jumping from one branch to another consequently decreases) with decreasing radius and/or latitude. For example, for curve 5 at 300 K the distance between the green and red branches is nearly 80 m s^{-1}, while for curve 2 it is about two times smaller. The transition point between the two branches for curve 5 corresponds to $T = -5^{\circ}$C, which is a temperature when the hurricanes do not form. The transition point moves to the region of lower temperatures with growing $r_p \sin\vartheta$. On the other hand, with increasing radius and/or latitude the area of allowed velocities becomes narrower. For example, for curve 1 at the observed temperatures only a narrow range of velocities in the vicinity of 46 m s^{-1} can be developed. These physical limitations shape the observed distributions of hurricane wind velocities over temperature, Fig. 4c.

Finally, when the angular momentum a decreases (due to either decreasing radius r_p of the condensation area as well as due to decreasing angular velocity ω at low latitudes), the eye radius decreases as well. The energy ot the eye rotation, which in the dimensionless variables (39) is equal to $a^2/4$, see (50), becomes small. In the result, there appears a possibility for tangential velocity in the windwall to grow to catastrophic values that are characteristic of tornadoes.

8 Angular momentum: the effect of superposition of several streamlines

Consideration of the energy budget of every streamline starts from $r = r_p$ and ends with $r = 0$, with the account made for the substraction of energy to sustain the eye rotation. Eye radius x_e for each streamline depends on angular momentum $a = r_p\omega/u_c$, i.e., it is also determined by the value of r_p. Angular momentum of the rotating air volumes in the eye changes with distance according to $ar^2/r_e^2 = ax^2/x_e^2$. Thefore, the complete information about a given streamline is contained in the value of angular momentum a that is conserved in the region outside the eye $x \geq x_e$.

Hurricanes apparently comprise a large number of individual streamlines, some of which can be easily traced on the satellite images as individual "tails" spiraling towards the hurricane center – the eye. These streamlines can start at various radii r_p and possess various values of angular momentum a that is conserved outside the eye.

The average angular momentum of the hurricane can be obtained considering the sum of kinetic energies of rotation for all streamlines as dependent on distance r to the circulation center. For each i-th streamline starting at $r = r_{pi}$ the dependence of angular momentum $a_i(x)$ and tangential velocity $v_i(x)$ on relative distance x has the form

$$a_i(x) = a_i \left[\frac{x^2}{x_{ei}^2} \vartheta(x_{ei} - x) + \vartheta(x - x_{ei})\vartheta(x_i - x)) \right], \quad v_i(x) = \frac{a_i(x)}{x},$$
$$x_i \equiv \frac{r_{pi}}{r_{p\,max}}, \; x \equiv \frac{r}{r_{p\,max}} \leq 1, \; a_i = \frac{r_{pi}\omega}{u_c}, \; \vartheta(X) \equiv \begin{cases} 1, & \text{if } X \geq 0 \\ 0, & \text{if } X < 0 \end{cases}. \tag{57}$$

Here x is defined relative to the streamline with maximum ultimate radius $r_{p\,max} \geq r_{pi}$ and maximum conserved angular momentum; $\vartheta(X)$ is the step function. Mean angular momentum $a(x)$ as dependent on x can be obtained from the following relationship:

$$a(x) = \frac{\sum_i a_i(x) v_i^2(x)}{\sum_i v_i^2(x)} = \frac{\sum_i a_i^3(x)}{\sum_i a_i^2(x)} \approx \frac{1}{n} \sum_i a_i(x), \tag{58}$$

where $a_i(x)$ and $v_i(x)$ are defined in (57). The last approximate equality in (58) is written from the condition $\sum_i a_i^2(x) \approx n a^2(x)$, $a_i^3(x) \approx a^2(x) a_i(x)$ to qualitatively evaluate the behavior of $a(x)$. As one can see from Eq. (58), the a_i values that enter the sum diminish with decreasing x. Hurricane eye acquires a complex structure that is formed by different streamlines at different radii x_{ei}. The closer to the center, the smaller values of angular momentum are that form the hurricane eye. These two facts testifiy that, in agreement with observations, $a(x)$ decreases towards the center despite the angular momentum is conserved within each particular streamline.

9 The pole approximation

The circulation system of hurricanes and tornadoes can be considered in the pole approximation. Such a consideration preserves the main physical features and the quantitative estimates of velocities and condensational potential. In the pole approximation one preserves the terms that display most singularity at $x \to 0$, namely, the major powers $1/x$ and $\ln x$. This is equivalent to discarding $\partial u/\partial x$ as being small compared to u/x in the Bernoulli integral (43) and in the Euler equation for the streamline that corresponds to differentiating both parts of (43) over x. In this case the Bernoulli integral (43) simplifies to become (Makarieva and Gorshkov, 2009b)

$$u^2 + v^2 + w^2 + \ln x = 0, \quad v^2 = \frac{a^2}{x^2}, \quad w^2 = \beta^2 \frac{u^2}{x^2}, \tag{59}$$

$$u^2 = \left(-\ln x - \frac{a^2}{x^2} \right) \frac{x^2}{x^2 + \beta^2}, \quad \beta \equiv \frac{h_\gamma}{r_p}, \quad p(x) = \ln x. \tag{60}$$

For hurricanes, where $\beta \ll 1$, Eq. (60) takes the form

$$u^2 = -\ln x - \frac{a^2}{x^2}. \tag{61}$$

The point of maximum $x_m = \sqrt{2}a$. For maximum radial velocity u_m^2 we have

$$u_m^2 = -\ln(\sqrt{2}a) - \frac{1}{2}. \tag{62}$$

Velocities u and w become strictly equal to zero at $x = x_0$ defined from the equation

$$-\ln x_0 - \frac{a^2}{x_0^2} = 0. \tag{63}$$

Tangential velocity reaches its maximum $v_e = a^2/x_e^2$ at $x_e = 1.28x_0$.

As one can see from the comparison of (41) and (59), the pole approximation consists in replacing condensational potential $\ln[(u/u_1)x]$ for $\ln x$, i.e., it does not depend on the initial velocity u_1. The gradient of the potential $1/x + u^{-1}\partial u/\partial x$ is replaced by $1/x$, i.e. the term containing $\partial u/\partial x$ is discarded. This changes the profile of the potential in the region from $x = 1$, $u = u_1 = u(1)$ to $x = x_0$, $u = u_0 = u(x_0)$, while the change of the potential (pressure difference δp) is not affected by the approximation. As far as with decreasing x the value of u first grows at $x \geq x_m = \sqrt{2}a$ and then declines at $x \leq x_m$, the exact potential declines more slowly than the potential in the pole approximation at $x \geq x_m$ and more rapidly than the latter at $x \leq x_m$.

Tornadoes, where $\beta \geq 1$, preserve the main features of the pole approximation. Angular momentum in the tornado is very small, $a = r_p\omega/u_c \geq h_\gamma\omega/u_c \leq 0.004$ (at $h_\gamma \sim 10$ km, $\omega \sim 3 \times 10^{-5}$ s, $u_c \sim 80$ m s^{-1}). Due to the fact that the eye in tornadoes is extremely narrow, $r_0 = h_\gamma x_0 = h_\gamma a/2 \sim 20$ m, the eye rotation practically does not take any energy from the streamline. Neglecting the difference between x_0 and x_e we have from Fig. 4a that $x_0 \sim x_0 \sim a/2$, see Fig. 6a. From (60) we have $\delta p \sim p(1) - p(x_0) = -\ln x_0 \sim 6$, Fig. 6b, so that $\delta p \Delta p/p \sim 6\gamma \sim 0.3$. In the center of the tornado the air pressure can drop by 30% compared to the outer environment. This causes tangential velocity to grow to very high values of the order of $(a/x_0)u_c \sim 2u_c \sim 160$ m s^{-1}, Fig. 6a. In contrast to the hurricane, while approaching $x = x_0$ the radial velocity u in tornado decreases proportionally to x^2, see (60) and Fig. 6a, and has a maximum at $x = 0.69$ ($-\ln x = 1/2.$). Vertical velocity w (59) in tornado has the same behaviour as radial velocity u in the hurricane, see (60). In the point of maximum $x_m = \sqrt{2}a$ the vertical velocity reaches $w_m = u_c(-ln(\sqrt{2}a) - 1/2)^{1/2} \sim 160$ m s^{-1}, see (62). This maximum value

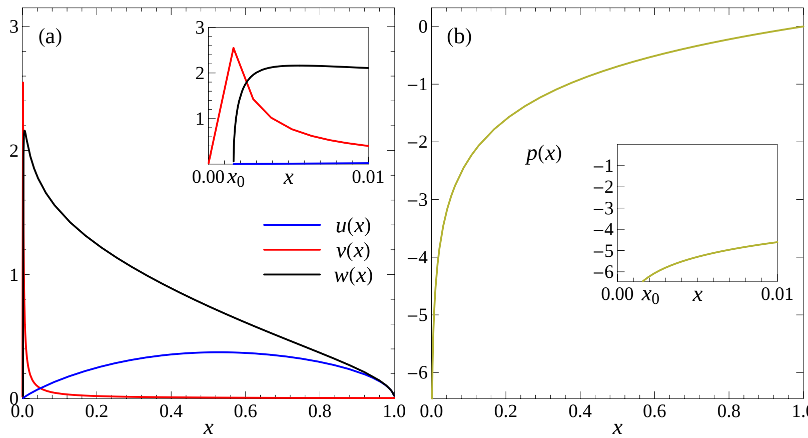

Figure 6: Wind velocities and pressure profile in a tornado calculated in the pole approximation.
(a): vertical w, tangential v and radial u velocities in units of $u_c = (2\Delta p/\rho)^{1/2}$ (38) depending on distance x to the condensation center (x measured in units of the condensational scale height h_γ (11) at $\beta = 1$) with dimensionless angular momentum $a = h_\gamma \omega / u_c$, x_0 is the eye radius that satisfies $\ln x_0 = a^2/x_0^2$, Typical values at 30°C are $u_c \sim 80$ m s^{-1}, $h_\gamma \sim 10$ km, Fig. 2, $\Delta p \sim 4 \times 10^3$ Pa, $a = 0.004$, $x_0 = 0.0016$.
(b): pressure fall $p(x)$ in units of Δp from the outer edge of the condensation area at $x = 1$ to the windwall – eye radius at $x = x_0$.

of vertical velocity coincides with the value of tangential velocity reached at $x_0 \sim a/2$, Fig. 6a. Radial velocity u in tornadoes changes much more slowly at small x than it does in hurricanes, $\partial u/\partial x \ll u/x$, which makes the pole approximation more exact for tornadoes than it is for the hurricanes.

We note one more peculiarity of the tornado. The eye radius is $x_0 \sim a/2$, Fig. 4, $a = h_\gamma \omega / u_c$, $u_c = (2\Delta p/\rho)^{1/2} = (2p/\rho)^{1/2}\gamma^{1/2}$, see (34). The water vapor relative partial pressure γ decreases approximately exponentially with growing height and practically turns to zero at 5–8 km, see (18). Therefore, the eye radius in the tornado grows proportionally to $\gamma^{-1/2}$ with increasing height z. This creates a conspicuous "mouth" of the tornado funnel – the funnel is narrow at the surface and widens as the height grows. This is another indication that tornado represents a single streamline in contrast to the hurricane, where the eye is formed as the result of many streamlines being superimposed.

10 Major features of atmospheric circulation induced by water vapor condensation

1. The physical cause of the condensation-induced circulation consists in the fact that hydrostatic equilibrium of the moist terrestrial atmosphere is unstable and cannot exist (Sections 1, 2).

In an isothermal atmosphere partial pressure of water vapor would exponentially decline with height decreasing twofold per each nine kilometers of height increment. According to Clausius-Clapeyron law, saturated water vapor would follow the same exponential distribution if the air temperature dropped by approximately ten degrees K per each nine kilometers of height increment, i.e., with a lapse rate of 1.2 K km^{-1}. However, adiabatic ascent of a volume of moist air causes air temperature to drop at a significantly higher rate bounded between approximately 4 and 9.8 K km^{-1}. Consequently, a random adiabatic displacement of moist air upwards makes the excessive water vapor condense and leave the gas phase. Pressure in the adiabatically ascending air drops. There appears an upward-direction evaporative-condensational force that sustains continuous adiabatic ascent of the moist air and causes air to circulate in the vertical and horizontal directions.

2. Stationarity of the annually averaged state of the atmosphere and the surface testify to the equality between the global mean rates of evaporation and condensation rates. However, evaporation is maintained by the stationary flux of solar radiation. Mean power of the evaporation flux cannot exceed the power of solar radiation. In contrast, the power of the flux of water vapor condensation is not related to the solar radiation power and can locally exceed the evaporation power by one-two orders of magnitude (e.g., in hurricanes and tornadoes) or become by the same factor less intense than local evaporation (in the regions of the anticyclonic descent of air masses). These contrasting properties of evaporation and condensation are crucial for the condensation-induced circulation.

3. In a large-scale horizontal circulation the air flows from the donor area to the acceptor area in the lower atmosphere and in the opposite direction in the upper atmosphere. In the donor region the

air masses descend; there is no condensation, but the evaporation is present. In the acceptor area the air masses ascend; there is condensation; both locally evaporated water vapor and the water vapor imported from the donor area undergo condensation. The rate of condensation is thus always higher in the acceptor area than in the donor area; this sustains the pressure gradient necessary for the maintenance of the air flow. Water returns to the donor area in the liquid phase. In a large-scale circulation total evaporation can coincide with total condensation (then the circulaton can be stationary in the long-term). However, the circulation itself is only possible due to the absence of equality between local evaporation and condensation (they do not coincide in either donor or acceptor area). Condensation rate in the acceptor area must be higher than in the donor area. This is stably achieved by higher evaporation rate in the acceptor area compared to the donor area.

4. In large-scale (Hadley, biotic pump of continental-scale natural forest) and medium-scale (moonsoon) circulation systems friction forces prevent significant acceleration of air masses and do not allow high wind speeds to develop. If there is no control over condensation and evaporation, the areas where such circulation systems develop are also prone to the appearance of frequent floods, droughts, tornadoes and hurricanes of varying intensity. The condensation-induced circulation can be regulated only with use of the genetically based biological programs of natural forest ecosystems, where the necessary soil moisture store is continuously maintained by biotic regulation of evaporation and condensation.

5. Compact circulation takes place where friction is small. Condensation power that locally exceeds the power of evaporation by many times accelerate the air to catastrophic velocities observed in hurricanes and tornadoes. In a compact circulation there is a pronounced condensation center, to which the rotating air masses converge. The condensation center moves continuously in the horizontal plane. This movement guarantees that the circulation system is supplied by new amounts of water vapor which is depleted as the system functions. If compact circulation do not move, they dissipate (similar to the movement of an animal over its feeding territory).

6. With increasing mixing ratio γ of water vapor in the moist air and as this ratio approaches unity due to the removal of the non-condensable air constituents (oxygen and nitrogen), the moist

adiabatic lapse rate approaches the hydrostatic value of 1.2 K $k,^{-1}$. The evaporative-condensational force that supports the adiabatic ascent tends to zero. The adiabatic ascent stops; the atmosphere becomes vertically isothermal; condensation-induced circulation systems of all types disappear. But with increasing temperature the evaporative-condensational force grows exponentially.

7. The condensation-induced circulation arises on horizontally isothermal surface and does not demand an external differential heating to arise.

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