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Condensation-induced dynamic gas fluxes in a mixture of condensable and non-condensable gases

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ABSTRACT

It is shown that condensation of water vapor produces dynamic instability of atmospheric air and induces air circulation that is characterized by observable air velocities and persists independently of the magnitude of horizontal temperature gradients.

1. Introduction

In this Letter we describe how dynamic fluxes are generated in a gas mixture containing a condensable gas in the presence of a vertical temperature gradient in the gravitational field. We will consider water vapor as condensable gas to retain the physical linkage to the terrestrial atmosphere. Condensation leads to disappearance of water vapor from the gas phase, which produces drop of local air pressure and creates a wind-inducing pressure gradient force that is proportional in magnitude to the amount of water vapor that undergoes condensation. In the presence of a sufficiently large vertical temperature gradient the vertical distribution of saturated partial pressure p_{H_2O} departs significantly from the static equilibrium; at any height p_{H_2O} is over five times larger than the weight of water vapor column above this height [1]. For this reason practically all water vapor ascending in the atmosphere undergoes condensation. The volume-specific store of potential energy available for conversion into kinetic energy of air movement can thus be estimated as the value of partial pressure p_{H_2O} of saturated water vapor to a good approximation.

2. Condensation-induced pressure gradient force

In contrast to dry air, atmospheric water vapor finds itself under the action of two independent physical factors, gravity and temperature. On the one hand, as all other (non-condensable) air gases, water vapor tends to the state of aerostatic equilibrium defined by the equation $\partial p_v/\partial z + \rho_v(z)g = 0$, where p_v is partial pressure of water vapor, ρ_v is its mass density, z is height above the planetary surface. Using the equation of state for ideal gas $\rho_v/M_v = p_v/(RT)$, $M_v = 18\ \mathrm{g\,mol^{-1}}$ is water vapor molar mass, $R = 8.3\ \mathrm{J\,mol^{-1}\,K^{-1}}$ is the universal gas constant, T is absolute temperature, the condition for aerostatic equilibrium of water vapor [1] can be written as

$$\frac{1}{p_v(z)}\frac{\partial p_v}{\partial z} = -\frac{1}{h_v(z)},$$
$$h_v(z) \equiv \frac{RT(z)}{M_v g}, \qquad h_v(0) = 13.5\ \mathrm{km}. \tag{1}$$

On the other hand, partial pressure of water vapor cannot exceed the saturated partial pressure $p_{H_2O}(z)$, $p_v(z) \leqslant p_{H_2O}(z)$, the latter depending on temperature $T(z)$ as dictated by Clausius–Clapeyron equation. This equation can be written in a form similar to that of Eq. (1):

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$$\frac{1}{p_{H_2O}(z)}\frac{\partial p_{H_2O}(z)}{\partial z} = -\frac{1}{h_{H_2O}(z)},$$

$$h_{H_2O}(z) \equiv h_v(z)\frac{\Gamma_{H_2O}(z)}{\Gamma}, \qquad h_{H_2O}(0) = 2.4\ \text{km}, \tag{2}$$

$$\Gamma \equiv -\frac{dT(z)}{dz}, \qquad \Gamma_{H_2O}(z) \equiv \frac{T(z)M_v g}{Q},$$

$$\Gamma_{H_2O}(0) \equiv \Gamma_{H_2O} = 1.2\ \text{K}\,\text{km}^{-1}. \tag{3}$$

Here $Q = 44\ \text{kJ}\,\text{mol}^{-1}$ is molar vaporization heat of water vapor (latent heat), $T(0) = 288$ K.

It follows immediately from Eqs. (1) and (2) that saturated water vapor can only be in aerostatic equilibrium when $h_v(z) = h_{H_2O}(z)$, i.e. $\Gamma = \Gamma_{H_2O} = 1.2\ \text{K}\,\text{km}^{-1}$ [1]. The critical value of Γ_{H_2O} determines the condition $\Gamma > \Gamma_{H_2O}$ when saturated water vapor cannot be in equilibrium. In this case the difference between the right-hand and the left-hand parts of Eq. (1) is not zero and represents an upward force f_E acting on a unit volume of moist air:

$$f_E(z) = -\frac{\partial p_{H_2O}(z)}{\partial z} - g\rho_{H_2O}(z) = \frac{p_{H_2O}(z)}{h_{H_2O}(z)}\left(1 - \frac{\Gamma_{H_2O}}{\Gamma}\right). \tag{4}$$

The observed mean vertical temperature gradient in the lower atmosphere corresponds to $\Gamma = 6.5\ \text{K}\,\text{km}^{-1}$, which is six times a larger value than Γ_{H_2O}, Eq. (3). Therefore the last term, Γ_{H_2O}/Γ, in brackets of Eq. (4) is only 1/6. The largest value of Γ_{H_2O}/Γ is observed in hurricanes [2] and constitutes $\Gamma_{H_2O}/\Gamma \sim 1/4$.

Work $A_E = \int_0^\infty f_E(z)\,dz$ performed by force f_E when raising a unit volume of moist air along the entire atmospheric column is, to the accuracy of a few per cent neglecting the change of absolute temperature in the lower part of the atmosphere up to h_{H_2O}, $[T(z) - T(0)]/T(0) \leqslant \Gamma h_{H_2O}/T(0) = 0.05$, equal to

$$A_E = f_E(0)h_{H_2O}(0) = p_{H_2O}(0)\left(1 - \frac{\Gamma_{H_2O}}{\Gamma}\right) = \Delta p \equiv \rho\frac{u_E^2}{2}. \tag{5}$$

Velocity u_E has the meaning of vertical velocity of air that has been accelerated by force f_E along the entire atmospheric column. Force f_E and work A_E are present everywhere in the atmosphere where there is saturated water vapor. Since force f_E arises due to condensation of water vapor in the atmosphere, which can only be sustained if there is a compensating process of evaporation from the hydrosphere, it is logical to term this force as the evaporative-condensational force. Since $\Gamma_{H_2O}/\Gamma = 1/6$, work A_E does not depend on latent heat to the accuracy of 17%, Eq. (5).

3. Large-scale stationary circulation, hurricanes and tornadoes

Consider two extensive horizontal regions where water vapor is present in the atmosphere in approximately equal quantities, but condensation in one region is more intense than in the other region. The ascending air flow caused by condensation in the first region leads to the inflow of moist air masses from the second region; the imported water vapor serves to sustain the condensation process in the first region. We denote the approximately equal lengths of the acceptor (first) and donor (second) regions for L, their width (equal to the length of the border between them) for D, vertical velocity of air masses ascending within the acceptor region for w, horizontal air velocity for u. The mass-conserving equality between the horizontal air flux entering the acceptor region via vertical atmospheric cross-section Dh_{H_2O} with velocity u (h_{H_2O} is the scale height of the vertical distribution of atmospheric water vapor, see Eq. (2), and, hence, describes the part of the atmospheric column where the evaporative-condensational force f_E is in action) and the vertical air flux leaving the acceptor region via horizontal atmospheric area DL (the integral continuity equation) reads as

$$Lw = h_{H_2O}u. \tag{6}$$

Note that h_{H_2O} approximately coincides with the atmospheric height below which the horizontal air velocity retains its direction.

Turbulent friction force f_T acting per unit air volume can be represented via friction force per unit surface area F_T that acts on atmospheric column of height h_{H_2O} where air moves in one and the same direction:

$$f_T = F_T/h_{H_2O}. \tag{7}$$

The circulation is stationary in strict terms when the rate of condensation is equal to the rate of evaporation sustained by solar power for any period of time and when the power of evaporative-condensational force f_E is equal to the power of turbulent friction force f_T. These conditions can be written for work A_E or power W_E of force f_E, see Eq. (5):

$$W_E = f_E\bar{w} = f_T\bar{u}, \qquad A_E = f_E h_{H_2O} = f_T L. \tag{8}$$

Here vertical velocity $w = \bar{w}$ is the average vertical velocity of upwelling moist air masses that corresponds to the mean evaporation rate from the surface, $\bar{w} \sim 10^{-3}\ \text{m}\,\text{s}^{-1}$ [1]. Using Eq. (8) for average spatially constant horizontal velocity $\bar{u}$ and linear dimension of the acceptor region $\bar{L}$ we have

$$\bar{u} = K\bar{w}, \qquad \bar{L} = Kh_{H_2O}, \quad K \equiv \frac{f_E}{f_T} = \frac{\bar{u}}{\bar{w}}. \tag{9}$$

Taking global mean values of $\bar{u} \sim 7\ \text{m}\,\text{s}^{-1}$ [3] and $\bar{w} \approx 1.3\ \text{mm}\,\text{s}^{-1}$ [1], we have $K \approx 5400$ in Eq. (9). Finally, using Eqs. (9) and (2), we have $L \sim 10^4$ km. This approximate estimate shows that, in order to enjoy a stable stationary circulation with a spatially constant velocity that is continuously sustained by evaporation and solar radiation, the donor and acceptor regions must be very large.

Let us consider the nature of resistance force F_T. Motion of terrestrial animals and transport involve two types of resistance forces. These are the friction force per unit surface area, which is independent of velocity and proportional to the weight of the body, and air resistance, which is proportional to the squared velocity of movement u. Out of dimensional considerations these two friction terms per unit surface area can be written in the following form [4]:

$$F_T = \mu\rho gh + C_D\rho u^2. \tag{10}$$

Here ρ is mass density of the body, $g = 9.8\ \text{m}\,\text{s}^{-2}$ is acceleration of gravity, h is the mean height of the body, $\mu \sim 1$ is the dimensionless coefficient of friction of rest, C_D is the dimensionless drag coefficient reflecting the geometry of the moving body [5, §45]. Drag coefficient C_D is of the order of unity for spherical bodies; in the general case it is proportional to the ratio of the vertical cross-section area of the body to the area of its projection on the Earth's surface, i.e. to ratio h/L, where h is height, L is length of the body [4].

In meteorology turbulent friction is formally represented by the second term of Eq. (10) with drag coefficient C_D being of the order of 10^{-3} [6]. If we apply an analogy to the motion of animals and use the continuity equation (6), C_D can be estimated, in its order of magnitude, as

$$C_D = ch_{H_2O}/L = cw/u, \tag{11}$$

where c is of the order of unity, i.e. as the ratio of the vertical to horizontal dimensions of the circulation pattern equal to the ratio of vertical to horizontal velocity.

A stationary constant value of velocity along some part of the streamline indicates absence of air acceleration, which corresponds to the equality between the pressure gradient force F_A that accelerates the air column and turbulent friction force F_T (7) that opposes the motion (forces are taken per unit area of the Earth's surface and have the dimension of $\text{N}\,\text{m}^{-2} = \text{J}\,\text{m}^{-3} = \text{Pa}$):

$$F_A \equiv \Delta p h_{H_2O}/\bar{L}, \qquad F_A = F_T. \tag{12}$$

Here Δp is the horizontal pressure difference observed over horizontal distance $\bar{L}$, $\Delta p/\bar{L}$ is the accelerating pressure gradient force per unit air volume, h_{H_2O} is characteristic height of the part of atmospheric column where horizontal wind retains its direction and the characteristic scale height of water vapor vertical distribution. Taking a characteristic horizontal pressure gradient on Earth to be in the order of $\Delta p/\bar{L} \sim 1$ mbar $(100\ \text{km})^{-1} = 1\ \text{Pa km}^{-1}$, height $h_{H_2O} \approx 2$ km [1], air density $\rho = 1.3\ \text{kg m}^{-3}$, global mean velocity $\bar{u} \sim 7\ \text{m s}^{-2}$ [3] and comparing F_A and F_T in Eq. (12) we have:

$$F_A \sim 2\ \text{N m}^{-2} \gg C_D \rho u^2 \sim 0.06\ \text{N m}^{-2}. \tag{13}$$

That is, the accelerating pressure gradient force exceeds the aerodynamic drag force by more than thirty times. This comparison shows that the account of aerodynamic turbulent friction $C_D \rho u^2$ is essentially insufficient for explaining the observed nearly constant horizontal velocities of the order of a few meters per second that can be maintained on the major part of the streamline over horizontal distances of the order several thousand kilometers [7]. For example, air circulation over the Amazon river basin is characterized by wind velocities $u \sim 5\ \text{m s}^{-1}$ maintained practically year round over distances of $\bar{L} \sim 2 \times 10^3$ km [8]. This implies that the air masses travel for several days inland at nearly constant velocity before they ascend and reverse the direction of their motion in the upper atmosphere. In the meantime, the force imbalance implied by Eq. (13) would correspond to an unrealistic acceleration $a \sim \Delta p/(L\rho) \sim 1.5 \times 10^{-4}\ \text{m s}^{-2} \approx 13\ \text{m s}^{-1}\ \text{day}^{-1}$ even if a conservative estimate of horizontal pressure gradient $\Delta p/L \sim 0.2\ \text{Pa km}^{-1}$ (10 hPa over 5 thousand kilometers) is applied.

The problem can be solved by the proposition that there is another term in the cumulative friction force F_T that is analogous to the first term in Eq. (10) and independent of horizontal velocity u. Total friction force can then be written as

$$F_T = F_{T0} + F_{Ta}, \quad F_{T0} = \rho g z_T,\ F_{Ta} = C_D \rho u^2. \tag{14}$$

Here F_{Ta} characterizes the aerodynamic turbulent friction in the lower part atmospheric column $z \leqslant h_{H_2O}$, while F_{T0} represents surface turbulent friction and z_T is the vertical scale of surface roughness that can slightly depend on velocity u.

Air pressure p at the Earth's surface is approximately equal to the weight of atmospheric column. According to the equation of state it can be written as $p = \rho g h = 10^5\ \text{J m}^{-3}$ $(1\ \text{J m}^{-3} = 1\ \text{N m}^{-2})$, $h = RT/Mg = 8.4$ km is the atmospheric exponential scale height, $M = 29\ \text{g mol}^{-1}$ is molar mass of air. Due to surface roughness, horizontal air movement leads to formation of turbulent eddies at the surface. The energy of these eddies is substracted from the kinetic energy of the mean horizontal air flow and the flow ceases to accelerate. Interaction of turbulent eddies with the surface and with each other leads to their dissipation into yet smaller eddies and ultimately to dissipation to heat [5]. Energy of the turbulent eddies is proportional to atmospheric pressure and to linear size z_T of surface roughness. By analogy to the first term in Eq. (10), F_{T0} can then be written as

$$F_{T0} = \rho g z_T = \mu \rho g h, \quad \mu \equiv z_T/h. \tag{15}$$

Since on the global average $F_T = F_A \gg C_D \rho u^2$, Eq. (12), this means that $F_A = \rho g z_T$. Eq. (12) in this case gives $z_T \sim 0.2$ m and $\mu \sim 2 \times 10^{-5}$. In other words, compared to the case of the motion of solid bodies along a rough surface that is characterized by $\mu \sim 1$, Eq. (10), surface turbulent friction for air moving along a rough surface is, as is well known, vanishingly small (i.e., if the atmospheric column of the same weight were a solid body, its movement along the Earth's surface would be characterized by $\mu \sim 1$ instead of $\mu \sim 2 \times 10^{-5}$). However, the absolute value of surface turbulent friction F_{T0} appears large enough for it to be the major source of resistance in large-scale circulation patterns where horizontal wind velocities of the order of several meters per second are maintained along the major part of the horizontal streamline of the order of 10^3 km:

$$F_A = F_T \approx F_{T0} \gg F_{Ta}. \tag{16}$$

The value of z_T represents an independent linear scale corresponding to the actual linear size of surface inhomogeneities. Therefore, if one chooses an area with z_T much smaller than the mean value for which Eq. (16) holds, then over such a surface term F_{T0} may become negligible. Then the only term that can be empirically measured on such an area will be F_{Ta} (see, e.g., [9] on C_D measurements on a smooth concrete field with $z_T \sim 10^{-3}$ m $\ll 0.2$ m).

We now note an essential detail. According to Eqs. (7), (8) and (5) the turbulent friction force is equal to $f_T = \Delta p/\bar{L}$. In the stationary case, when the horizontal acceleration is absent, pressure gradient force should be equal to turbulent friction force. This means that pressure difference Δp is uniformly distributed along the horizontal part of the streamline with a constant pressure gradient equal to $\Delta p/L = f_T = \rho g z_T/h_{H_2O}$. As the horizontal part of the streamline is thousand of times longer than the vertical part, $\bar{L} \sim 10^3 h_{H_2O}$, the non-equilibrium pressure difference along the vertical part of the streamline is significantly smaller than the total pressure difference Δp.

Change of horizontal velocity u along the streamline l is determined by Euler equation

$$\frac{1}{2}\rho\frac{\partial u^2}{\partial l} + \frac{\partial p}{\partial l} + f_T = 0. \tag{17}$$

Here $f_T = F_T/h_{H_2O}$ (7) is the mean force of turbulent friction acting on unit air volume in the part of atmospheric column $z \leqslant h_{H_2O}$.

Taking into account that along the entire streamline $\Delta p \ll p \approx \rho g h$, one can neglect the change of air density ρ along the streamline putting $\partial p/\partial l = \Delta p/L$, where L is length of the horizontal part of the streamline. We can now determine horizontal dimension $L = \bar{L}$ of such a circulation pattern where acceleration is absent $(\partial u^2/\partial l = 0)$ and velocity $u = \bar{u}$ is constant over the major part of the streamline. We thus have

$$\bar{L} = h_{H_2O}\frac{\Delta p}{F_T}, \quad \text{or} \quad \bar{L} = h_{H_2O}\frac{\bar{u}}{\bar{w}}, \quad \bar{u} = \bar{w}\frac{\Delta p}{F_T}, \tag{18}$$

where $\bar{w}$ is the mean vertical velocity of ascending air that can be determined from the rate of precipitation.

In circulation patterns where length L is smaller than $\bar{L}$ (18), $L \ll \bar{L}$, Eqs. (12) and (18) do not hold. From Eq. (17) we then obtain the following relationship for the increase of radial velocity u characteristic for the hurricane:

$$\frac{1}{2}\rho\frac{\partial u^2}{\partial l} = -\frac{\Delta p}{L} \gg f_T, \quad \text{or} \quad u^2 = \frac{\Delta p}{L}\frac{2l}{\rho}, \quad l \leqslant L_0 < L. \tag{19}$$

Radial velocity ceases to grow $(\partial u^2/\partial l = 0)$ at a distance $l = L_0$, where the aerodynamic resistance F_{Ta} (14), (11) grows up to F_A (12). In this case from Eq. (17) we have

$$\frac{\Delta p}{L} = \frac{F_{Ta}}{h_{H_2O}} = c\rho\frac{u^2}{L_0}, \quad \text{or} \quad u^2 = \frac{\Delta p}{L}\frac{L_0}{c\rho}, \quad L_0 \leqslant l < L. \tag{20}$$

This should coincide with the second equality in Eq. (19) at $l = L_0$, $(L - L_0)/L \ll 1$. From this condition $c = 1/2$ in Eq. (11) is derived giving $C_D = (1/2)h_{H_2O}/L$ in good agreement with observations [6]. Eq. (20) means that the aerodynamic turbulent friction (14), (11) coincides with the accelerating pressure gradient force and sets the limit to the increase of wind velocity in the hurricane.

When horizontal size L diminishes to the values of the order of atmospheric height h_{H_2O}, there appears a possibility of tornado formation with maximum possible velocity u and $w \sim u$. The simplest case, when work A_E is released and the locally accumulated potential energy per unit volume $p_{H_2O} \approx \Delta p$ is converted to kinetic energy $\rho u_E^2/2$, is the case when intense condensation of water vapor occurs in a local area surrounded by dry areas where water vapor concentration is very low. In this case the ascending air flow in the condensation area sucks in dry air masses from the neighboring areas. The converging air streams approach the condensation area with some non-zero angular momenta, which inevitably results in the spiral-like rotating structure of hurricane and tornado with the vortex funnel of the eye where air pressure additionally drops due to the centrifugal force. This structure is therefore an immediate consequence of the vertical force f_E acting in the three-dimensional space.

References

[1] A.M. Makarieva, V.G. Gorshkov, Hydrol. Earth Syst. Sci. 11 (2007) 1013.
[2] B.I. Miller, J. Meteorol. 15 (1958) 184.
[3] M.R. Gustavson, Science 204 (1979) 13.
[4] V.G. Gorshkov, Zh. Obsch. Biol. 44 (1983) 661.
[5] L.D. Landau, E.M. Lifshitz, Course of Theoretical Physics, 6, Fluid Mechanics, 2nd ed., Butterworth-Heinemann, Oxford, 1987.
[6] J.R. Garratt, Mon. Wea. Rev. 105 (1977) 915.
[7] B.I. Miller, Mon. Wea. Rev. 92 (1964) 389.
[8] J. Zhou, K.-M. Lau, J. Clim. 11 (1998) 1020.
[9] P.A. Sheppard, Proc. R. Soc. London A 188 (1947) 208.