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Condensation-induced kinematics and dynamics of cyclones, hurricanes and tornadoes

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ABSTRACT

A universal equation is obtained for air pressure and wind velocity in cyclones, hurricanes and tornadoes as dependent on the distance from the center of the considered wind pattern driven by water vapor condensation. The obtained theoretical estimates of the horizontal profiles of air pressure and wind velocity, eye and wind wall radius in hurricanes and tornadoes and maximum values of the radial, tangential and vertical velocity components are in good agreement with empirical evidence.

1. Introduction

Condensation of water vapor in the upper low-temperature part of the atmosphere creates a drop of local air pressure. Wet air is then sucked into the acceptor region where condensation takes place from the donor regions where condensation is less intense or absent. Water vapor that formed during evaporation within the donor regions is brought to the acceptor region in the lower part of the atmosphere. It then undergoes condensation within the rising air flow. This process sustains and enhances the initial pressure drop of air pressure in the acceptor region. Deprived of water vapor by condensation, the relatively dry air in the upper part of the atmosphere flows towards the donor region, where the air descends. The resulting circulation pattern which length is of the order of several thousand kilometers was quantitatively considered in [1,2].

Based on consideration of condensation dynamics here we describe radially symmetric patterns of atmospheric circulation, namely the wind velocity and pressure fields of cyclones, hurricanes and tornadoes. The maximum radius of such patterns is estimated to be 1500 km. If the process of condensation affects larger areas, a radially symmetric circulation pattern is destroyed at its very birth by the component of turbulent surface friction that is independent of velocity [2]. In this case only a linear circulation pattern between the donor and acceptor regions can form characterized by constant wind velocities [2].

2. Radially symmetric wind pattern

We first consider an inertial frame of reference with a cylindrical coordinate system, where condensation occurs within a horizontal circle of radius L. As shown in [2], change of air pressure due to water vapor condensation is

$$\Delta p = p_{H_2O}\left(1 - \frac{\Gamma_{H_2O}}{\Gamma}\right) \equiv \rho\frac{u_m^2}{2}, \quad u_m^2 \equiv \frac{2\Delta p}{\rho}. \tag{1}$$

Here Δp_{H_2O} is partial pressure of water vapor, $\Gamma_{H_2O} = 1.2\ \text{K}\,\text{km}^{-1}$ and Γ are the critical and observed negative vertical gradients of air temperature, respectively. At $\Gamma = \Gamma_{H_2O}$ saturated water vapor is in aerostatic equilibrium in the entire atmospheric column and there is no condensation. Velocity u_m represents the main wind velocity scale in the considered problem.

When water vapor undergoes condensation within a circle of radius L, the air pressure drops. Air starts to flow via circumference of radius $r < L$ towards the center with radial velocity u and

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simultaneously rises with vertical velocity w. Air flows horizontally via the round vertical wall of circumference $2\pi r$ and height h and rises via the horizontal circle of area πr^2. In the integral form the continuity equation then reads

$$2\pi hru(r) = 2\pi \int_0^r r' w(r')\, dr'. \tag{2}$$

Here h is the scale height along which condensation of water vapor occurs. Velocity $w(r)$ represents mean vertical wind velocity. To simplify the calculations, we neglected the horizontal change of air density with r, as it does not exceed 5% in most circulation patterns observed in the atmosphere. By differentiating Eq. (2) over r we find that $u(r)$ and $w(r)$ are related as

$$w(r) = \frac{h}{r} u(r)\left(1 + \frac{r}{u(r)}\frac{du(r)}{dr}\right). \tag{3}$$

The horizontal drop of air pressure as the air flows towards the center is determined by condensation of water vapor that occurs as the air rises. The rates of these processes coincide, so $\frac{dp}{dr}u(r) = \frac{\Delta p}{h} w(r)$. Using (3), from this we obtain the following key relationship:

$$\frac{dp}{dr} = \frac{\Delta p}{h}\frac{w(r)}{u(r)} = \Delta p\left(\frac{1}{r} + \frac{1}{u}\frac{du}{dr}\right). \tag{4}$$

The dependence of radial velocity u on distance r from the center is determined by Euler equation for the radial streamlines that go inward towards the center of the circulation pattern:

$$\frac{1}{2}\rho\frac{d(u^2+w^2)}{dr} = -\frac{dp}{dr} + f_T, \quad f_T = C_D\frac{\rho(u^2+w^2)}{h} + \rho g z_T \frac{1}{h},$$
$$u^2 \geqslant 0, \quad u^2(L) = 0. \tag{5}$$

Here f_T is the turbulent friction force acting per unit air volume, dimension $\mathrm{N\,m^{-3}} = \mathrm{J\,m^{-4}}$ [2]. It is equal to the sum of the aerodynamic component that is proportional to squared total velocity and the velocity-independent component that describes the effect of surface roughness [2]; $C_D = (1/2)h/L$ is the aerodynamic resistance [2], z_T is the roughness scale of the Earth's surface.

We will neglect the contribution of vertical velocity w to the aerodynamic resistance, as w is smaller than u in most circulation patterns, and discard terms w^2 compared to u^2 in the left-hand part of Eq. (5). Accounting for dw^2/dr is only important in very compact circulation patterns, namely tornadoes with $L \leqslant h$. As can be seen from Eq. (3), retaining this term yields an equation that contains $(du/dr)^2$, i.e. is non-linear with respect to the first derivative of u.

The whole circulation pattern is contained within the circle of radius L, where condensation occurs. This determines the boundary condition $u^2(L) = 0$. We multiply both parts of Eq. (5) by $2/\rho$ and go over to dimensionless variables using the length and velocity scales L and u_m (1). Eqs. (4) and (5) then yield the following non-linear equation:

$$\frac{dy}{dx} = -\frac{1}{x} - \frac{1}{2y}\frac{dy}{dx} + y + c, \quad y \geqslant 0, \quad y(1) = 0, \tag{6}$$

$$y \equiv \frac{u^2}{u_m^2}, \qquad x \equiv \frac{r}{L}, \qquad c \equiv 2\left(\frac{g z_T}{u_m^2}\right)\frac{L}{h}, \qquad \Delta p \equiv \rho\frac{u_m^2}{2}. \tag{7}$$

Eq. (6) can be re-written as

$$\frac{dy}{dx} = \left(-\frac{1}{x} + y + c\right)\frac{2y}{2y+1}. \tag{8}$$

Here the second (non-linear) multiplier in the right-hand part of the equation derives from the second term in the right-hand part of Eq. (6). Note that Eq. (6) has a polar term x^{-1}, which is a consequence of the polar symmetry of the considered problem, see Eqs. (3) and (4). This term leads to a logarithmic divergence at $x \to 0$. This divergence persists irrespective of the magnitude of turbulent friction described by the last two terms in Eq. (6). This divergence is physically relaxed by a radical drop of air pressure and practical disappearance of the gas from the center of the circulation pattern.

3. Account of the Earth's rotation

Due to the Earth's rotation with radial velocity Ω the air masses at the border of the radially symmetric circulation pattern at $r = L$ rotate at angular frequency $\omega_L = \Omega \sin\vartheta$, $\Omega = 1.16 \times 10^{-5}\ \mathrm{s^{-1}}$, where ϑ is the latitude angle counted from the equator, with tangential velocity $v_L = \omega_L L$. In this case the air masses are affected by the centrifugal force $f_c = \rho\frac{v^2}{r}$, where v is tangential velocity that is perpendicular to radius r, f_c is radially directed away from the center; and by the Coriolis force $f_C = 2\rho\omega u$ that is perpendicular to radius r. The aerodynamic component of turbulent friction is proportional to squared total velocity $u^2 + v^2$, which means that a tangential friction term $f_a = C_D\rho v^2/h$ should be added to the right-hand parts of Eqs. (5), (6). This component of the aerodynamic friction force is, similarly to the Coriolis force, perpendicular to radius r. Air motion under the action of radially symmetric forces conserves the angular momentum. The condition for angular momentum conservation takes the form $rv(r) = Lv_L$, where $v_L \equiv v(L) = \omega_L L$. The centrifugal force f_c appears then when the term $\rho(dv^2/dr)/2$ is added to the left-hand side of Eq. (5) (containing the derivative of total velocity $V^2 \equiv u^2 + v^2 + w^2$) and moved afterwards to the right-hand side of that equation. Under the condition of angular momentum conservation forces f_c, f_a and f_C conform to the following proportionality relationships $f_c \sim v^2/r \sim r^{-3}$, $f_a \sim v^2 \sim r^{-2}$, $f_C \sim v \sim r^{-1}$, $f_a/f_c \sim x$, $f_C/f_c \sim x^2$. Consequently, at $x \ll 1$ the non-radial forces — the tangential component f_a of turbulent friction and the Coriolis force f_C — are small in comparison to the radial centrifugal force f_c. This means that the angular momentum is conserved with a high accuracy. In terms of the dimensionless variables of Eqs. (7), (8), forces f_c, f_a and f_C when multiplied by $2/\rho$ take the form $\frac{2a^2}{x^3}$, $\frac{a^2}{x^2}$ and $\frac{a}{x}$, respectively, $a^2 \equiv \frac{v_L^2}{u_m^2}$. Adding f_c to the right-hand part of Eq. (6) and neglecting f_a and f_C we obtain:

$$\frac{dy}{dx} = -\frac{1}{x}\left(1 - 2\frac{a^2}{x^2}\right) - \frac{1}{2y}\frac{dy}{dx} + y + c, \quad a \equiv \frac{v_L}{u_m} \ll 1, \tag{9}$$

$$y \geqslant 0, \quad y(1) = 0. \tag{10}$$

4. Solving Eq. (9) in the polar approximation

The behaviour of $y \equiv \frac{u^2}{u_m^2}$ in Eq. (9) is dictated by the first two polar terms and by the last constant term c. Retaining these terms in Eq. (9) we obtain the following equation and its solution:

$$\frac{dy}{dx} = -\frac{1}{x}\left(1 - 2\frac{a^2}{x^2}\right) + c, \tag{11}$$

$$y(x) = -\ln x - a^2\left(\frac{1}{x^2} - 1\right) - c(1-x). \tag{12}$$

Using relationships (3) and (4) and definitions (7) we have:

$$u(x) = u_m\left[-\ln x - a^2\left(\frac{1}{x^2} - 1\right) - c(1-x)\right]^{1/2}, \tag{13}$$

$$w(x) = \frac{h}{L}\frac{u(x)}{x}, \tag{14}$$

$$v(x) = \frac{v_L}{x}, \quad v_L \equiv v(1), \tag{15}$$

$$\frac{1}{\Delta p}\frac{dp}{dx} = \frac{1}{x}, \qquad \frac{p(1)-p(x)}{\Delta p} = -\ln x, \tag{16}$$

$$x \equiv \frac{r}{L}, \qquad a \equiv \frac{v_L}{u_m}, \quad v_L \equiv \omega_L L, \quad u_m^2 \equiv \frac{2\Delta p}{\rho}. \tag{17}$$

5. The wind wall and the eye of hurricanes and tornadoes

Conservation of angular momentum causes tangential velocity v to increase with decreasing radius r. The physics of polar terms consists in the fact that potential energy of the gas released due to water vapor condensation $(dp/dx)/\Delta p$ is spent on the increment of the kinetic energy of the radial du^2/dx and tangential dv^2/dx air fluxes and partly lost to friction $(-c)$.

Kinetic energy of the tangential air flux grows with decreasing radius as r^{-3}. It grows more rapidly than does the rate of potential energy release during condensation, which is proportional to r^{-1}. Therefore at a certain distance determined by solution (12) all the potential energy released goes to increase the tangential kinetic energy. In the result, the increment of the radial kinetic energy turns to zero. According to (11) and (12) distance $r_m = x_m L$ where it happens is determined from the condition that the first derivative of y (11) is zero and the second derivative is negative. At $x = x_m$ radial velocity u and kinetic energy y (12) are maximum. Putting the right-hand side of (11) equal to zero we find

$$cx_m^3 - x_m^2 + 2a^2 = 0, \tag{18}$$

$$x_m \approx \sqrt{2}a, \quad a \ll 1, \quad c \leqslant 1. \tag{19}$$

At $x = x_m$ the derivative of y (11) changes its sign, so at $x < x_m$ the radial u and vertical w velocities drop rapidly from their maximum value to zero. They become zero at $x = x_e$, which is determined from Eq. (12):

$$-\ln x_e = \frac{a^2}{x_e^2} + c. \tag{20}$$

At $x \leqslant x_m$ the polar behaviour of function y changes little with or without accounting for the other (non-polar) terms in Eq. (9). Maximum tangential velocity is reached at $x = x_e$ and, according to (16), (17), is equal to

$$v_{\max} = v(x_e) \equiv v_e = u_m \frac{a}{x_e}, \quad a \equiv \frac{v_L}{u_m}. \tag{21}$$

Pressure gradient grows proportionally to r^{-1} from the outer border $r = L$ of the circulation pattern until $r = r_e$. Total pressure drop along the radial streamline from $r = L$ to $r = r_e$ is equal to

$$p_L - p_e \equiv \Delta p_{\text{out}} = \Delta p \int_{x_e}^{1} \frac{dx}{x} = \Delta p(-\ln x_e) > \Delta p. \tag{22}$$

At $x = x_e$ the tangential velocity v of air masses converging from the outskirts to the center of the circulation pattern is sustained not only by the potential energy of condensation but mainly by the expenditure of the kinetic energy of radial movement that has accumulated by $x = x_m$. Further acceleration of total kinetic energy of the air masses towards the center $x = 0$ becomes impossible. Point $x = x_e$, $r = r_e$ corresponds to the eye radius of hurricanes and tornadoes.

Within the eye the movement of air masses and the drop of air pressure are not determined by the process of condensation, as potential energy related to condensation is used up completely as the air masses reach $x = x_e$. The hurricane eye is characterized by sunny weather.

Within the eye at $x < x_e$ the centrifugal force exceeds the condensation-related force of the centripetal acceleration, the polar terms cancel each other and the turbulent friction remains the main term. Under the action of turbulent friction the air masses within the eye start to rotate at maximum angular velocity $\omega_e = v_e/r_e$ that is reached at $x = x_e$. Unlike in the solid body there are no centrifugal forces in the gas to compensate for the centripetal forces in the eye. Therefore the atmospheric air in the eye, including cloudiness, starts to leave the eye first to $r > r_e$ and then to the outer environment with $r > L$. This creates an additional drop of air pressure within the eye that continues to develop until the funnel is formed and the resulting pressure gradient compensates the centrifugal force. This corresponds to the following relationship, see (20):

$$\frac{dp}{dr} = \rho\frac{v^2}{r} = \rho\omega_e^2 r, \qquad p(r) - p(0) = \frac{1}{2}\rho\omega_e^2 r^2,$$
$$\omega_e = \frac{v_e}{r_e}, \quad v = w_e r, \tag{23}$$

$$p_e - p_0 \equiv \Delta p_{\text{in}} = \rho\omega_e^2 \int_0^{r_e} r\,dr = \rho\frac{v_e^2}{2} = \Delta p \frac{a^2}{x_e^2}$$
$$= \Delta p(-\ln x_e - c). \tag{24}$$

Comparing (22) and (24) one can see that the radial pressure change from the circulation border $r = L$ to $r = r_e$ and within the eye from $r = r_e$ to $r = 0$, Δp_{out} and Δp_{in}, respectively, coincide at $c = 0$ (which is the case of tornadoes) and exceed the value of Δp (1). For hurricanes at $c = 1$ the magnitude of Δp_{in} is about twice less than Δp_{out} (22).

The eye does not have a strict border. Its border can be defined as the area between x_e and x_m, where the angular velocity of the eye's rotation is distributed from $\omega_e = v_e/r_e$ to $\omega_m = v_m/r_m$. We can define the mean eye radius $\overline{x_e}$ demanding that pressure gradients (16) and (23) coincide at $x = \overline{x_e}$. This condition gives $\overline{x_e} = \sqrt{2}a \approx x_m$. At $a = 0.1$ and $c = 1$, as follows from (18), $\overline{x_e} = \sqrt{2}a$ is 10% smaller than the exact value of x_m determined by (18). If we join the pressure functions defined by (16) and (23) at $x = x_m$ with $w_m = v_m/r_m$, then for Δp_{out} and Δp_{in} we have:

$$\Delta p_{\text{out}} = -\Delta p \ln x_m,$$
$$\Delta p_{\text{in}} = \rho\frac{v_m^2}{2} = \frac{\Delta p}{2}, \quad x_m = \sqrt{2}a. \tag{25}$$

The obtained values are smaller than in (22) and (24), while $\Delta p_{\text{tot}} = \Delta p_{\text{out}} + \Delta p_{\text{in}} = 2.5\Delta p$. (At $a \leqslant 0.1$ we have $\Delta p_{\text{out}} \geqslant 2\Delta p$, $\Delta p_{\text{tot}} \geqslant 2.5\Delta p$.)

Kinetic energy of the radial and tangential air flows, $\rho u^2/2$ and $\rho v^2/2$, respectively, and the pressure drop profile (16), (23) are shown in Fig. 1.

6. Maximum dimensions of the radially symmetric circulation patterns

The air masses accelerate if only the turbulent friction forces are smaller than the pressure gradient force induced by water vapor condensation [2]. Turbulent aerodynamic friction is essential at large wind velocities only in linear circulation patterns [2]. Outside the eye the radially symmetric atmospheric circulation pattern does not depend on turbulent aerodynamic friction at any wind velocities, see Sections 3, 4. Turbulent friction is thus determined by turbulent surface friction described by coefficient c (7) that does not depend on velocity, but does depend on radius L of the circulation pattern. According to Eqs. (11), (12), the critical value is $c = 1$. At $c < 1$ there are solutions (11) that satisfy (10). At $c > 1$ there is no solution, which means that the turbulent friction is so

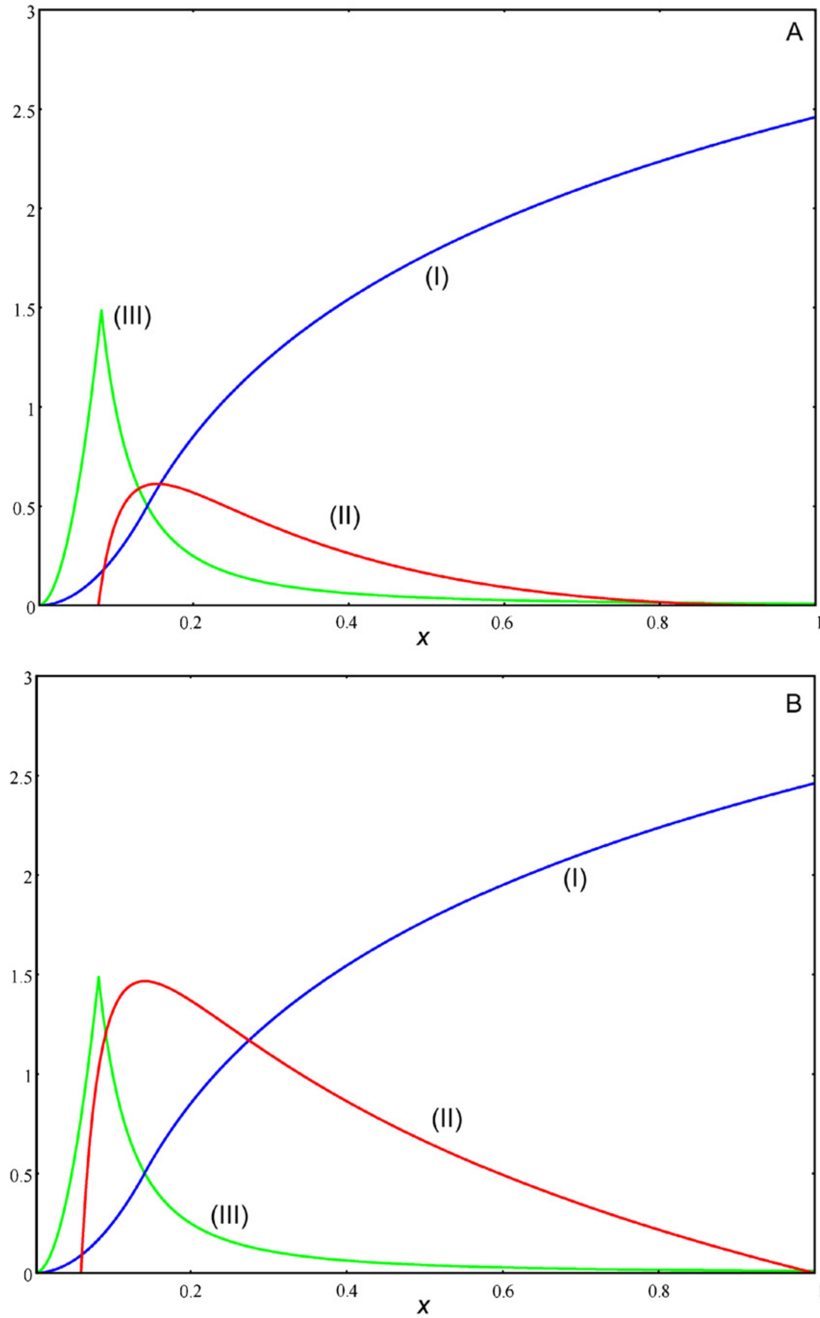

Fig. 1. Spatial structure of the radially symmetric circulation. Shown are the horizontal profiles of air pressure drop (I), Eqs. (14) and (23) were joined at $x = x_m$, see Eq. (25); kinetic energy of the radial (II) and tangential (III) air flows. Kinetic energy of the vertical air flow $\rho w^2/2$, see Eq. (14), is not shown as w is much smaller than u and v. On the horizontal axis $x = r/L$, r is distance from the center, L is the circulation radius. On the vertical axis all the variables are shown in Δp units, $\Delta p = \gamma p = 0.75\rho \frac{u_m^2}{2}$, see Eq. (1), $\gamma = p_{H_2O}/p = 0.05$, $p = 10^5$ Pa, $\rho = 1.3$ kg m^{-3}, $u_m = 76$ m s^{-1}. A: Hurricanes of maximum size, $L = 500$ km, turbulent surface friction $c = f_T/\Delta p = 1$, surface roughness $z_T = 1.4$ m. B: Tornadoes, $L = 20$ km, $c = 0$, $z_T < 5$ cm.

high that the air masses cannot accelerate and the wind does not enhance. The radially symmetric circulation pattern starts to develop in the boundary region $x = 1$, $r = L$. If the derivative of y is negative, $y = u^2/u_m^2$ is also negative at $x < 1$, which is physically impossible.

Condition $c = 1$ corresponds, according to (7), to the following equality:

$$c_{\max} = 2\left(\frac{gz_T}{u_m^2}\right)\frac{L_{\max}}{h} = 1, \quad L_{\max} = h\frac{1}{2}\frac{u_m^2}{gz_T}. \tag{26}$$

For the global mean values at $p_{H_2O}/p = 0.02$, $\Gamma_{H_2O}/\Gamma = 0.18$ we have at $u_m = 50$ m s^{-1}, $z_T = 0.2$ m and $h = 2.4$ km that $L_{\max} =$ 1500 km. The radially symmetric atmospheric circulation patterns (cyclones) cannot spread over a distance $2L$ exceeding 3×10^3 km. This conclusion agrees well with observations.

We now estimate the main characteristics of values (19)–(24) that characterize hurricanes and tornadoes considering inequality $c \leqslant 1$ as the condition for the origin of these wind structures. Other parameters specified, this condition stipulates the value of surface roughness z_T. The maximum wind velocities obtained at $c = 1$ represent minimal values. The decrease of $c < 1$ diminishes turbulent friction and elevates the maximum velocity values. For $\gamma = p_{H_2O}/p = 0.05$ (temperature 30 °C), $\Gamma_{H_2O}/\Gamma = 0.25$ we obtain from (1) $u_m = 76$ m s^{-1}. At latitude $\vartheta = 30°$ we have

$\omega_L = 0.6 \times 10^{-5}$ s^{-1}. Most values in (19)–(24) depend on parameter $a = v_L/u_m$. When the weather is absolutely still, $v_L = \omega_L L$, which gives $v_L = 3$ m s^{-1} for hurricanes with $L = 500$ km. However, for the birth of most hurricanes and all tornadoes the global mean wind speed of $\bar{v}_L = 7$ m s^{-1} [3] is more relevant. The angular momentum appears due to the difference in wind direction at the opposite parts of the circle of radius L where the circulation develops. Accordingly, all subsequent estimates are made for $a \equiv \bar{v}_L/u_m = 0.1$ (20). Maximum of radial velocity $u(x)$ is reached at $x_m = \sqrt{2}a = 0.14$, where tangential velocity becomes $v(x_m) \equiv v_m = \frac{u_m}{\sqrt{2}} = 54$ m s^{-1}. For hurricanes at $c = 1$ ($z_T = 1.4$ m) we have $u(x_m) = 35$ m s^{-1}, $w(x_m) = 1.2$ m s^{-1}, $x_e = 0.082$, $v_e = v_m \frac{x_m}{x_e} = 92$ m s^{-1}. Radius r_m of the maximum radial velocity and radius x_e of the eye at $L = 500$ km are $r_m = Lx_m = 70$ km, $r_e = Lx_e = 41$ km. The pressure drops become $\Delta p_{\text{out}} = 2.5\Delta p$, $\Delta p_{\text{in}} = 1.5\Delta p$, $\Delta p_{\text{tot}} = 4\Delta p$ if functions (22) and (24) are joined at $x = x_e$. If these functions are joined at $x = x_m$ we have $\Delta p_{\text{out}} = 2\Delta p$, $\Delta p_{\text{in}} = 0.5\Delta p$, $\Delta p_{\text{tot}} = 2.5\Delta p$. At $c = 0.5$ ($z_T = 0.7$ m) we obtain, respectively, $x_e = 0.067$, $u_m(r_m) = 73$ m s^{-1}, $v_e = 112$ m s^{-1}, $r_e = 34$ km, $r_m = 70$ km. All these values agree well with the available data on characteristic parameters of hurricanes (e.g., [4]).

In tornadoes that feature $L \sim h$ vertical velocity w (3) can exceed radial velocity u. Eqs. (6), (10) and (11) where the input of vertical velocity is neglected are not accurate at $L \sim h$. Therefore, the results obtained here for tornadoes are valid for $L \geqslant 10h \geqslant 20$ km, a stipulation to which most of the observed tornadoes conform. Putting $L = 20$ km, which corresponds to $c = 0.04 \ll 1$, and $a = 0.1$ we have: $x_m = 0.14$, $x_e = 0.060$, $u_{\max}(x_m) = 111$ m s^{-1}, $w_{\max}(x_m) = 95$ m s^{-1}, $v_{\max} = v(x_e) = 127$ m s^{-1}, $r_m = 2.8$ km, $r_e = 1.2$ km, $\Delta p_{\text{out}} = \Delta p_{\text{in}} = 2.8\Delta p$, $\Delta p_{\text{tot}} = 5.6\Delta p$, which agrees with observations [5,6]. If the tornado develops in the initially completely still environment we have $v_L = 0.12$ m s^{-1}, $a = 1.6 \times 10^{-3}$, $x_m = 2.2 \times 10^{-3}$, $x_e = 0.59 \times 10^{-3}$, $r_m = 32$ cm, $r_e = 12$ cm, all velocities reach supra-sound values near the eye, while air pressure in the eye drops to zero. This is caused by the singularity in the center of the non-rotating circulation pattern, see Section 2, that appears at $v_L \to 0$.

We also estimate velocity U which describes movement of the circulation pattern as a whole. The store of condensation-related potential energy is spent within the circle of diameter $2L$ in time $h/\overline{w}$, where $\overline{w}$ is the mean vertical wind velocity over the entire circle. The store of potential energy within the circle remains constant, if the circle moves with velocity U along the gradient of increasing water vapor concentration and covers diameter $2L$ in time $h/\overline{w}$, i.e. when the equality holds $\frac{2L}{U} = \frac{h}{\overline{w}}$, which gives $U = 2\overline{w}L/h$. The value of $\overline{w}$ can be obtained by integrating $w(x)$ over x in the vicinity of the main velocity peak at $x \sim x_m$, $x \sim x_e$ for $\Delta x \sim a$. This yields $\overline{w} \sim \frac{h}{2L}\bar{u}$, $\bar{u} \sim u_m a$, i.e. $U \sim u_m a \sim 7$ m s^{-1}. This agrees with the observable velocities of hurricane movement. For tornadoes with $L \sim 10h$ we have $U \sim \bar{u} \geqslant 100$ m s^{-1}, i.e. in any given place such a tornado would last for three minutes.

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