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Radial profiles of velocity and pressure for condensation-induced hurricanes

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ABSTRACT

The Bernoulli integral in the form of an algebraic equation is obtained for the hurricane air flow as the sum of the kinetic energy of wind and the condensational potential energy. With an account for the eye rotation energy and the decrease of angular momentum towards the hurricane center it is shown that the theoretical profiles of pressure and velocity agree well with observations for intense hurricanes. The previous order of magnitude estimates obtained in pole approximation are confirmed.

1. Introduction

In this Letter we obtain more exact solutions to the equations of the condensational theory of large atmospheric vortices (hurricanes and tornadoes) formulated in [1]. We first discuss that the equation for condensational potential [1, Eq. (4)] can be derived from the continuity equation where condensation rate for adiabatically ascending air in hydrostatic equilibrium is taken into account [2].

The fact that the hurricane air masses accelerate to high velocities indicates that the friction forces are small compared to the force of the condensational pressure gradient. This leads to the existence of the Bernoulli integral for radial streamlines. We show that this integral has the form of an algebraic equation on radial and tangential velocities under the condition that the vertical velocity is small (which is the case in hurricanes).

As the moist air ascends due to the condensation-induced pressure drop, a certain part of the angular momentum of the flow is exported upwards and away from the condensation area. In the result, the mean angular momentum in the flow decreases towards the hurricane center. The energy of the steady-state eye rotation not accounted for in [1] arises at the expense of the condensational potential energy, which decreases the maximum wind speed in the windwall. Considering these two processes, the export of angular momentum and eye rotation, we obtain realistic wind and pressure dependencies on distance from the center for axis-symmetric condensation-induced hurricanes.

We estimate the key theoretical parameters, including height h of the condensation area, the degree β of depletion by condensation of the local store of atmospheric vapor, the conserved part α of angular momentum that remains in the flow at the windwall, and compare the theoretical radial profiles to observations on most intense Atlantic hurricanes. We discuss how the changes in these parameters impact the hurricane intensity.

2. Condensational pressure gradient

Condensation rate S ($\mathrm{mol\,m^{-3}\,s^{-1}}$) for adiabatically ascending moist air can be formulated as follows [2]

$$S \equiv w\left(\frac{\partial N_v}{\partial z} - \frac{N_v}{N}\frac{\partial N}{\partial z}\right) \equiv wN\frac{\partial \gamma}{\partial z},$$
$$\gamma \equiv \frac{N_v}{N}, \quad N = N_v + N_d. \qquad (1)$$

Here $w > 0$ is vertical velocity, z is height; N, N_v and N_d are molar densities ($\mathrm{mol\,m^{-3}}$) of moist air, saturated vapor and the dry air component, respectively. Condensation rate S is proportional to the difference between the total change of N_v with height (the first term in brackets) and the change that is unrelated to condensation and affects all gases equally (adiabatic expansion, the second term in brackets). According to the Clausius–Clapeyron law, N_v depends on temperature only and does not change over an isothermal surface.

The stationary continuity equation in the cylindrical coordinates is

$$\frac{\partial N_v w}{\partial z} - \frac{1}{r}\frac{\partial N_v ur}{\partial r} = S,$$
$$\frac{\partial N w}{\partial z} - \frac{1}{r}\frac{\partial Nur}{\partial r} = S, \quad r < r_1,\ z < h. \qquad (2)$$

Here r is radial distance from the center of the condensation area, $u > 0$ is radial velocity directed towards the center, r_1 and h are,

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respectively, the radius and the height of the condensation area. At $z = h$ the relative humidity drops and condensation ceases. (Partially depleted of moisture, the air starts moving away from the condensation area back to the outer environment at $z > h$.) There is no dependence on angle φ in (2) due to the assumed radial symmetry.

Using (1) in (2) for N_v over an isothermal surface when $\partial N_v/\partial r = 0$ we obtain

$$\frac{1}{r}\frac{\partial ur}{\partial r} = \frac{1}{N}\frac{\partial Nw}{\partial z} \equiv \frac{w}{h}, \qquad h^{-1} \equiv h_w^{-1} - h_N^{-1},$$
$$h_w^{-1} \equiv \frac{1}{w}\frac{\partial w}{\partial z}, \qquad h_N^{-1} \equiv -\frac{1}{N}\frac{\partial N}{\partial z}. \tag{3}$$

Using Eq. (3) in Eq. (2) for N we obtain:

$$-u\frac{\partial N}{\partial r} = S. \tag{4}$$

Using the ideal gas law $p = NRT$, where R is the universal gas constant, T is absolute temperature, from (1) and (4) we obtain

$$h_\gamma u\frac{\partial p}{\partial r} = wp_v, \tag{5}$$

$$h_\gamma^{-1} \equiv h_v^{-1} - h_e^{-1} \equiv -\gamma^{-1}\frac{\partial \gamma}{\partial z},$$
$$h_v^{-1} \equiv -\frac{1}{p_v}\frac{\partial p_v}{\partial z}, \qquad h_e^{-1} \equiv -\frac{1}{p}\frac{\partial p}{\partial z}. \tag{6}$$

Putting w (3) into (5) we finally have

$$\frac{\partial p}{\partial r} = \Delta p\frac{\partial \ln ur}{\partial r}, \qquad p(r) = p(r_1) + \Delta p \ln ur, \tag{7}$$

$$\Delta p \equiv \beta p_v, \qquad \beta \equiv \frac{h}{h_\gamma} \approx \frac{\gamma(0) - \gamma(h)}{\gamma(0)}. \tag{8}$$

Scale height h_w (3) is determined by the finite size of the condensation area; h_γ is determined by the Clausius–Clapeyron law for h_v and the condition of hydrostatic equilibrium, $\partial p/\partial z = -NMg \equiv -p/h_e$, for h_e [2]. From $(1/p)\partial p/\partial z \gg (1/T)\partial T/\partial z$ we have $h_e = RT/Mg \approx h_N$, where $M \approx 29\ \mathrm{g\,mol^{-1}}$ is air molar mass. The value of $\beta \leqslant 1$ reflects that not all water vapor has condensed when the ascending air reaches height h. Eq. (7) coincides with Eq. (4) in [1].

3. The Bernoulli integral for hurricanes

The air masses accelerate towards the center of condensation area when the friction force is small compared to the pressure gradient force (7). In the absence of friction, the Bernoulli integral exists for the Euler equations along the streamline. It describes conservation of the sum of kinetic and potential energy in the flow, with potential energy equal to pressure p (7):

$$B(r) \equiv \frac{1}{2}\rho\left(u^2 + v^2 + w^2\right) + \Delta p \ln ur = B(r_1). \tag{9}$$

Here u, w and v are the radial, vertical and tangential (azimuthal) velocities, $\mathbf{v}$ is perpendicular to $\mathbf{u}$ and $\mathbf{w}$, ρ is air density that changes little at condensation due to $\gamma \equiv N_v/N \ll 1$ (1), r_1 is the radius of the condensation area ($S < 0$ at $r < r_1$). For tangential velocity we have $v = a/r$, where $a = a(r)$ is the angular momentum (per unit air mass). When $v \gg u$, the Coriolis force caused by the Earth's rotation is central and the angular momentum of the air that streams inward the condensation area is, in the absence of friction, conserved. Due to the Earth's rotation, the air has angular velocity $\omega = \Omega \sin\vartheta$, where ϑ is the latitude where the hurricane occurs, $\Omega = 2\pi/\tau$, $\tau = 24$ h. Angular momentum associated with Earth's rotation is equal to ωr^2, where r is the distance to the center of condensation area.

The condensation area and pressure field of the hurricane move as a whole with the so-called translation speed U. At any moment of time the hurricane flow represents a bunch of streamlines that converge towards the condensation center and ascend. Some of these streamlines (the "external" air) originate at or outside the border of the condensation area at $r \geqslant r_1$. While the hurricane moves over a given area, the external air which enters at $r = r_1$ and accelerates towards the center does not fully replace the local air. As a streamline of external air that has been partially depleted of vapor ascends somewhere at $r < r_1$ and leaves the condensation area, its place in the flow is taken by a streamline of local air that originates at that point, Fig. 1. The local air has a smaller angular momentum than the external air, but possesses the full store of water vapor. This ensures that water vapor remains on average saturated, the condition $\partial N_v/\partial r = 0$ fulfilled, and that the Bernoulli integral and air density remain the same for all the streamlines notwithstanding the export of the air mass, kinetic energy and angular momentum away from the condensation area with the outflow.

Sucking in the air from the lower atmosphere, the hurricane is surrounded by areas where the air is descending, the relative humidity is less than unity and there is no condensation. If there were no ascending motion within the hurricane until the air reached the windwall $r = r_0 \ll r_1$, then the angular momentum of the horizontally incoming air would be conserved and independent of r [1]. In this case all condensation would be concentrated at the windwall. In reality, relative humidity is lower than unity at the surface in the boundary layer of height $z_H < h$. At $z_H < z < h$ at any $r < r_1$ there are ascending motions present that export the angular momentum upwards away from the condensation area, Fig. 1. In the result, the mean angular momentum decreases towards the hurricane center. A certain part $\alpha \sim z_H/h$ of the original angular momentum is conserved: it is associated with the air that penetrates at $z < z_H$ from $r = r_1$ to the windwall $r_0 \ll r_1$, Fig. 1. This part can be estimated from the height z_H where the relative humidity approaches unity and condensation commences.

Total angular momentum $a(r)$ can thus be represented as the sum of two terms, one of which is constant and another one diminishes towards the center:

$$a(r) = a_{in} + a_{out}(r), \quad a_{in} \equiv \alpha a(r_1). \tag{10}$$

Here $a_{out}(r)$ describes the decrease of angular momentum with decreasing radius that is proportional to the change of vertical velocity $w(r)$ over r.

We now go over to dimensionless variables

$$\frac{p}{\Delta p}, \quad \frac{r}{r_1}, \quad \frac{u}{u_c}, \quad \frac{w}{u_c}, \quad \frac{v}{u_c}, \quad \frac{a}{u_c r_1}, \quad u_c^2 \equiv \frac{2\Delta p}{\rho}, \tag{11}$$

while retaining for them the original notations p, r, u, w, v, a, respectively. This is equivalent to choosing the units of measurements

$$r_1 = u_c = \Delta p = \rho/2 = 1. \tag{12}$$

To find $a_{out}(r)$ we integrate (3) over r in the new dimensionless variables (12):

$$rhu = \int_{r_0}^{r} w(r')r'\,dr', \qquad hu_1 = \int_{r_0}^{1} w(r')r'\,dr', \quad u_1 \equiv u(1). \tag{13}$$

Here r_0 is the hurricane eye radius, where condensation ceases, see the next section. The relative number of streamlines that originate at $r \geqslant 1$ and remain in the condensation area at a given radius r, Fig. 1, can be approximated by the ratio of the first to the second

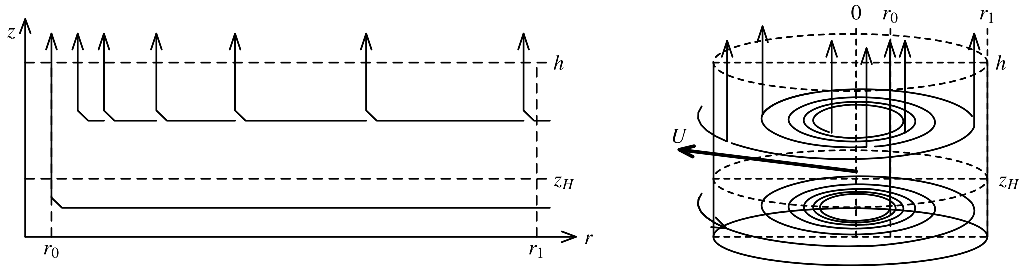

Fig. 1. The proposed scheme of hurricane streamlines. U is the so-called translation speed – the velocity of hurricane movement as a whole.

integrals in (13) that is equal to ru/u_1. Thus, the angular momentum $a_{out}(r)$ that remains in the flow at r is

$$a_{out}(r) = a_{out}\frac{ur}{u_1}. \qquad (14)$$

Condensation is accompanied by two processes: the air pressure falls in the condensation area, while the induced air flow tends to equate pressures inside and outside the hurricane. The relaxation velocity u_R is proportional to the relative pressure change caused by condensation, $\beta\gamma$, and the sound speed c in gases, $u_R \sim \beta\gamma c \sim 6\ \text{m}\,\text{s}^{-1}$. The relaxation velocity significantly exceeds the vertical velocity w in hurricanes. Therefore, despite condensation, an approximate hydrostatic equilibrium is established in the hurricane, such that at each r the condensational pressure drop is uniformly vertically distributed along the atmospheric column within the hurricane. In the horizontal dimension the condensational pressure gradient should become noticeable when the radial velocity is not smaller than the relaxation velocity u_R. Radial velocity at the outer border of the condensation area should coincide with u_R. It then grows with increasing condensational pressure gradient and drops back to u_R in the hurricane eye at $r = r_0$, where condensation ceases. We can write the Bernoulli integral (9) as follows:

$$B(r) - B(1) = u^2 - u_1^2 + v^2 - v_1^2 + \ln\frac{ur}{u_1} + w^2 - w_1^2 = 0, \qquad (15)$$

$$v = \frac{a}{r}, \quad a = a_{in} + a_{out}\frac{ur}{u_1},$$
$$w = \frac{h}{r}\frac{\partial ur}{\partial r}, \qquad p(r) = p_1 + \ln\frac{ur}{u_1}, \qquad (16)$$
$$u_1 \equiv u(1) = u_R, \qquad v_1 = a_1 \equiv a(1),$$
$$w_1 \equiv w(1), \qquad p_1 \equiv p(1). \qquad (17)$$

Condensation starts at $r = 1$ and $u = u_1 = u_R$ and ceases at small $r_0 \ll 1$, where radial velocity diminishes back to $u_1 = u_R$. Taking into account that the eye radius in hurricanes is of the order of 20 km [3], while condensation height h is about 4 km, one can neglect the vertical velocity terms $w^2 - w_1^2$ in (15) as minor terms of the order of $r_0^2/h^2 \sim 0.04$. Then the Bernoulli integral for hurricanes turns into an algebraic equation on the radial velocity u:

$$u^2 - u_1^2 + \frac{a^2}{r^2} - a_1^2 + \ln\frac{ur}{u_1} = 0. \qquad (18)$$

From (18) we observe that the eye radius where $u(r_0) = u_1$ is determined by the following equation:

$$-\ln r_0 = \frac{a^2(r_0)}{r_0^2} - a_1^2. \qquad (19)$$

4. Hurricane eye

The conserved part of angular momentum a_{in} (10) is one of the main hurricane parameters. At $r < r_0$ the vertical and radial velocity, as well as the condensational pressure gradient, disappear. As follows from (19), with decreasing a_{in} the eye radius diminishes, while the tangential kinetic energy of the windwall, $a_{in}^2/r_0^2 \approx -\ln r_0$, as well as the maximum wind velocity, grow. (This growth corresponds to concentration, in a progressively diminishing windwall area, of total condensational potential energy βp_v accumulated within the condensation area at $r < r_1$.)

The air in the eye rotates with a constant angular velocity ω_0 that coincides with the angular velocity of the windwall. For the eye rotation to be steady the centrifugal force must be compensated by the radial pressure gradient (the so-called cyclostrophic balance):

$$\frac{2v^2}{r} = \frac{\partial p}{\partial r}, \qquad \rho = 2 \quad \big[\text{see (12)}\big],$$
$$v^2 = \omega_0^2 r^2, \qquad p(r) = \omega_0^2 r^2 + \text{const}. \qquad (20)$$

The kinetic energy of eye rotation and potential energy of the pressure gradient within the eye (these two energies are equal) arise at the expense of the tangential kinetic energy of the windwall. This subtracts some energy from the windwall and reduces the maximum wind speed. For simplicity let us introduce a finite width of the windwall $\Delta r = r_e - r_0$ and assume that at $r_0 \leqslant r \leqslant r_e$ the tangential velocity is constant. Then the outer radius r_e of the windwall, where tangential velocity reaches its maximum $v = v_e$, is obtained from the following equation:

$$2\int_0^{r_0} v_0^2 r\,dr = \int_{r_0}^{r_e} \big(v^2 - v_e^2\big) r\,dr,$$
$$v_0 = \frac{a(r_e)r}{r_0 r_e}, \quad v_e = \frac{a(r_e)}{r_e}, \quad v = \frac{a(r_e)}{r}. \qquad (21)$$

Solving (21) gives

$$\ln\frac{r_e}{r_0} = \frac{1}{2}, \quad r_e = 1.65 r_0, \qquad (22)$$

where r_0 is obtained from (19).

We can now write out the complete radial profile of tangential velocity and pressure:

$$v(r) = \frac{\tilde{a}(r)}{r},$$
$$\tilde{a}(r) = \begin{cases} a(r) = a_{in} + a_{out} ru/u_1, & r_e < r < 1, \\ a(r_e) r/r_e, & r_0 \leqslant r \leqslant r_e, \\ a(r_e) r^2/r_0 r_e, & 0 \leqslant r \leqslant r_0; \end{cases} \qquad (23)$$

$$\delta p(r) \equiv p(1) - p(r) = \begin{cases} -\ln(ur/u_1), & r_0 < r < 1, \\ \omega_0^2(r_0^2 - r^2) - \ln r_0, & 0 \leqslant r \leqslant r_0; \end{cases} \qquad (24)$$

$$\delta p \equiv \delta p(0) = \omega_0^2 r_0^2 - \ln r_0, \quad \omega_0 = \frac{a(r_e)}{r_0 r_e}. \qquad (25)$$

Here $u(r)$ and $\delta p(r)$ are obtained solving (18) for $r_0 \leqslant r \leqslant 1$ with $a(r)$ given by (16). The pressure fall $\delta p(r)$ within the eye is obtained from (20) demanding that the pressure profile has no discontinuity at $r = r_0$; $\delta p \geqslant 0$ is the total pressure difference between the outer border of the condensation area $r = 1$ and the hurricane center $r = 0$. The first and second terms in (25) describe the pressure fall within and outside the eye, respectively. Note that at large a_{in} and r_0 (19), when $\omega_0^2 r_0^2 \gg -\ln r_0$, the hurricane should turn into a usual cyclone.

5. Comparison to observations

Height h of the condensation area can be estimated as the altitude beyond which the relative humidity starts rapidly decreasing upwards. This indicates the predominance of dry air and absence of condensation. Below we take $h = 4.5$ km that corresponds to pressure $p(h) \approx 600$ mb, where the average relative humidity in hurricanes drops from 0.8–0.9 at $p(z) > p(h)$ in the lower atmosphere to 0.4–0.6 at $p(z) < p(h)$ [4]. Ascending up to this height from the surface with temperature $T_s = 303$ K, the moist saturated air has lost to condensation $\beta \approx 0.4$ (8) of its total vapor content [2, Fig. 4d]. Saturated pressure of water vapor at $T = 303$ K is $p_v = 4.2 \times 10^3$ Pa. This gives the relaxation velocity $u_R = \beta(p_v/p_s)c = 6\ \mathrm{m\,s^{-1}}$, where $p_s \equiv p(0) = 10^5$ Pa and $c = 340\ \mathrm{m\,s^{-1}}$ is the sound velocity. Note that $\Delta p \equiv \beta p_v$ (8) represents the mean volume-specific store of condensational potential energy in the condensation area.

The conserved part α of the angular momentum can be roughly estimated by considering that due to hurricane movement with translation speed U the total influx of moist air into the condensation area of radius r_1 is proportional to $2r_1hU$ (the air is delivered to the moving condensation area via its cross-section $2r_1h$). To actually enter the hurricane area, the air descends and is rapidly sucked in via a circular wall of height z_H with inflow rate $2\pi r_1 z_H u_1$. Equating the two fluxes we have $\alpha \approx z_H/h = (U/u_1)(1/\pi)$. Observing that a characteristic translation speed $U \sim 4$–$7\ \mathrm{m\,s^{-1}}$ [5], from $u_1 \sim U$ we can estimate $\alpha \sim 1/\pi \approx 0.3$. This estimate can be obtained in a different way from the condition that at $z = z_H$ the air is saturated. For relative humidity $R_H \equiv p_v/p_n < 1$ we have

$$\frac{1}{R_H}\frac{\partial R_H}{\partial z} = \frac{1}{p_n}\frac{\partial p_n}{\partial z} - \frac{1}{p_v}\frac{\partial p_v}{\partial z} = \frac{1}{h_v} - \frac{1}{h_e} \equiv \frac{1}{h_H}. \qquad (26)$$

Here p_n is the non-saturated pressure of water vapor, which is assumed to have the same scale height $h_e = RT/Mg$ (6) as the non-condensable atmospheric gases. The value of h_v is determined by Clausius–Clapeyron law and the vertical temperature gradient [2, 6], at $T_s = 303$ K we have $h_e = 8.8$ km and $h_v = 5$ km [2, Fig. 4b]. This gives $h_H = 11.6$ km. We obtain height z_H from the condition that $R_H(z_H) = 1$: $z_H = -h_H \ln R_H(0)$. At $R_H(0) = 0.85$ we have $z_H = 1.5$ km and $\alpha \sim z_H/h = 0.4$. Note that thus defined α decreases with increasing relative humidity at the surface.

In Fig. 2 the velocity and pressure profiles (23)–(25) are shown for $h = 4.5$ km, $\beta = 0.4$, $T_s = 303$ K, $\alpha = 0.4$, $r_1 = 420$ km, $a(r_1) = \Omega r_1^2 \sin\vartheta = 4.4 \times 10^6\ \mathrm{m^2\,s^{-1}}$, $\vartheta = 20°$, Fig. 2A, and compared to the data on Category 5 (most intense) hurricanes [3]. Mean (± st. dev.) values for the eye radius REYE, radius of maximum wind RMW, maximum wind speed MW recorded for the recent twelve Category 5 hurricanes during their peak intensity are REYE $= 14 \pm 7$ km, MWR $= 28 \pm 9$ km, and MW $= 74 \pm 3\ \mathrm{m\,s^{-1}}$. These compare satisfactorily to the corresponding theoretical values of $r_0 = 19.6$ km, $r_e = 32.4$ km, and $v(r_e) = 72\ \mathrm{m\,s^{-1}}$, Fig. 2B. Note, however, that r_0 and r_e are not by definition equivalent to the eye radius and radius of maximum wind speed due to the approximation $v(r) = \text{const}$ at $r_0 \leqslant r \leqslant r_e$ adopted when building the theoretical profiles (23)–(25).

The mean observed pressure drop (the difference in surface pressure between the outer ambient environment p_a and hurri-

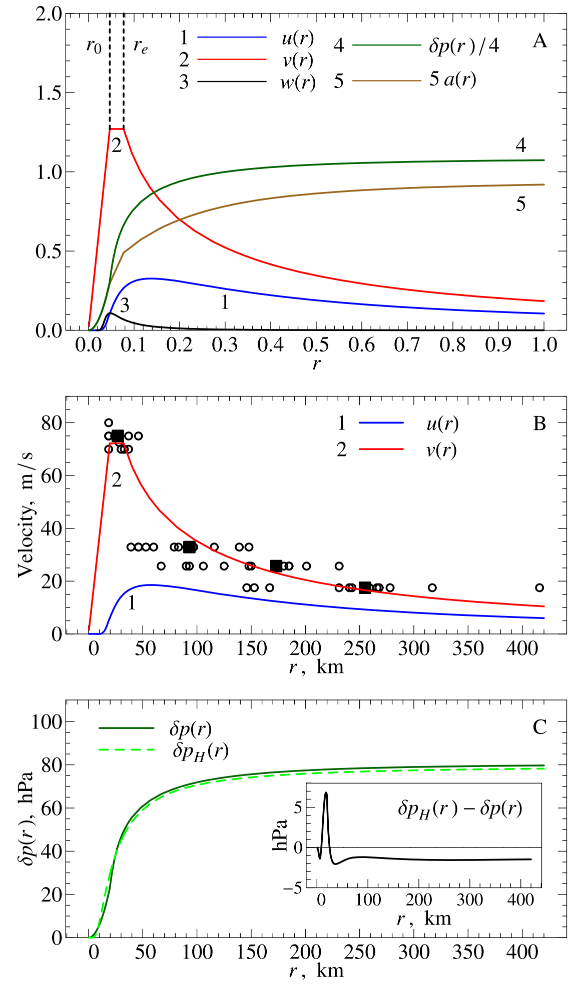

Fig. 2. Profiles of radial $u(r)$, tangential $v(r)$, vertical $w(r)$ velocities, angular momentum $a(r)$ and pressure $\delta p(r)$ in intense hurricanes. A: Profiles (23)–(25) in dimensionless variables (11). Note that $a(r)$ and $\delta p(r)$ are scaled. B: Same as in A for the radial and tangential velocities, but in dimensional variables, using $u_1 = 6\ \mathrm{m\,s^{-1}}$, $r_1 = 420$ km, $\Delta p = 18.5$ hPa, $u_c = 57\ \mathrm{m\,s^{-1}}$, see text. Open circles denote data for twelve most recent Category 5 hurricanes from the extended best track dataset [7]. Points correspond to the mean radii where a given wind speed (maximum velocity, 32.9, 25.7 or 17.5 $\mathrm{m\,s^{-1}}$, respectively) was observed (12 points are shown for each speed). Squares correspond to the median values for Category 5 hurricanes as given in Fig. 20 of [3]. Mean latitude of observations was $20 \pm 4°$. Radius $r_1 = 420$ km was taken to be equal to the maximum radius of gale force wind speeds (17.5 $\mathrm{m\,s^{-1}}$) observed in the twelve hurricanes. Temperature $T_s = 303$ K was chosen as the temperature corresponding to maximum observed hurricane wind speeds [8]. C: Same as in A for pressure, but in dimensional variables. Also shown is the empirical Holland's profile with $b = 1.3$, see text. Studied hurricanes (name, month date hour, year): Felix 090300 2007, Hugo 091518 1989, Mitch 102712 1998, Emily 071700 2005, Wilma 101912 2005, Ivan 091118 2004, Dean 082106 2007, Gilbert 091318 1988, Isabel 091318 2003, Rita 092200 2005, Andrew 082318 1992, Katrina 082818 2005, see ftp://rammftp.cira.colostate.edu/demaria/ebtrk/ebtrk_readme.txt.

cane center p_c) in the twelve hurricanes was $p_a - p_c = 95 \pm 15$ hPa (1 hPa $= 10^2$ Pa) compared to the theoretical value of $\delta p = 80$ hPa obtained from (25). The obtained δp agrees well with the corresponding pressure difference at height $z = z_H$. Taking $p(z) = p(0)\exp(-z/h_e)$, where $p(0)$ is surface pressure, at $z_H = 1.5$ km and $h = 9$ km we have $\delta p(z_H) = (p_c - p_a)\exp(-z_H/h) = 80$ hPa.

To compare the shape of the theoretical pressure profile to observations we employed the widely used empirical Holland's profile [9,10] $\delta p_H(r) \equiv \delta p \exp(-\mathrm{RMW}/r)^b$. Here δp is the observed pressure drop, RMW is the observed radius of maximum wind speed and b is a fitting parameter. Taking a characteristic $b = 1.3$

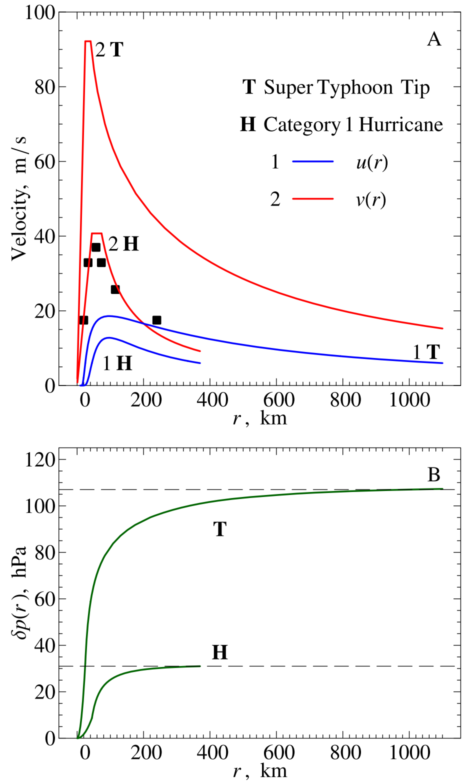

Fig. 3. Velocity (A) and pressure (B) profiles for a Category 1 hurricane and Super Typhoon Tip [12]. For Category 1 hurricane $u_1 = 6\ \mathrm{m\,s^{-1}}$, $r_1 = 370$ km, $\Delta p = 11$ hPa, $u_c = 43\ \mathrm{m\,s^{-1}}$; for Super Typhoon Tip $u_1 = 6\ \mathrm{m\,s^{-1}}$, $r_1 = 1100$ km, $\Delta p = 20$ hPa, $u_c = 59\ \mathrm{m\,s^{-1}}$. Squares correspond to the median values for Category 1 hurricanes as given in Fig. 21 of [3].

from empirical profiles fitted on intense hurricanes [11, Fig. 17], and using the theoretical values $\delta p = 80$ hPa and RMW $= r_0$, we plotted $\delta p(r)$ (25) versus $\delta p_H(r)$. The difference between the two profiles does not exceed 7 hPa, i.e., 9% of δp, at any r, Fig. 2C.

The key parameters can be modified to describe a wide range of vortexes, from relatively weak to superintense. In Fig. 3 the theoretical velocity and pressure profiles are contrasted for an average Category 1 hurricane and Super Typhoon Tip (the ever largest tropical cyclone with the ever lowest minimal surface pressure [12]). For the Category 1 hurricane we took $h = 2.5$ km and $\beta = 0.3$, $T_s = 300$ K, $\alpha = 0.8$, $r_1 = 370$ km (the mean radius of the outermost closed isobar [3, Fig. 20]), $a(r_1) = \Omega r_1^2 \sin\vartheta$, $\vartheta = 20°$. These parameters yielded maximum tangential velocity $v(r_e) = 41\ \mathrm{m\,s^{-1}}$, pressure drop $\delta p = 31$ hPa, $r_0 = 45$ km and $r_e = 74$ km in approximate agreement with observations [3,13], Fig. 3. For Super Typhoon Tip we took $h = 4.5$ km and $\beta = 0.4$ as for Category 5 hurricanes, $T_s = 305$ K, $\alpha = 0.1$, $r_1 = 1100$ km (the maximum radius where 15 $\mathrm{m\,s^{-1}}$ winds were reported [12]), $a(r_1) = \Omega r_1^2 \sin\vartheta$, $\vartheta = 11°$. These parameters yielded maximum tangential velocity $v(r_e) = 92\ \mathrm{m\,s^{-1}}$ (a value close to the maximum of 107 $\mathrm{m\,s^{-1}}$ ever recorded in a hurricane [14]) and a pressure drop of $\delta p = 107$ hPa, $r_0 = 25$ km and $r_e = 41$ km. The available observations describe Tip as having 85 $\mathrm{m\,s^{-1}}$ maximum velocity and maximum pressure drop 130 hPa at the surface [12].

Other parameters kept constant, the decrease of maximum intensity (smaller maximum wind speed and pressure drop) corresponds to smaller h (and, hence, lower β), lower temperature (hence lower p_v and Δp) and lower relative humidity at the sea level (hence, higher z_H and higher α). Possible interrelationships of these parameters and the hurricane translation speed U are to be investigated.

6. Dissipation of power in steady-state hurricanes

Hurricane power $W(r)$ ($\mathrm{W\,m^{-2}}$) is determined by the rate at which the condensational potential energy is converted to the kinetic energy of wind as specified by (5). The left-hand side of (5) represents the condensational pressure gradient force that acts on a circular wall of height h_γ and compresses the air inside the wall towards the center at a speed u. The right-hand part of (5) describes the total pressure drop of condensing vapor that makes the air ascend at vertical velocity w. Complete condensation of vapor in the atmospheric column corresponds to surface precipitation ($\mathrm{mol\,m^{-2}\,s^{-1}}$) equal to $P(r) = N_v w(r)$. From (5) we have $W(r) = w(r)p_v = P(r)RT$. Total hurricane power W_{tot} due to condensation of βp_v (8) is obtained by integrating (5) with use of $\partial T/\partial r = 0$ in ordinary (dimensional) variables from r_0 to r_1:

$$W_{tot} = 2\pi h \int_{r_0}^{r_1} u\left(-\frac{\partial p}{\partial r}\right) r\,dr = 2\pi h r_1 p u_b = P_{tot} RT,$$

$$P_{tot} = 2\pi \int_{r_0}^{r_1} N_v w r\,dr, \qquad u_b \equiv \gamma u_1 = W_{tot}/(pV_{tot}),$$

$$p_v = \gamma p = N_v RT, \qquad V_{tot} = 2\pi r_1 h. \tag{27}$$

As can be seen from (27), the power at which the potential energy of condensation is converted to the kinetic energy of the air flow is, on the one hand, equal to the power at which the condensation area surrounded by a circular wall of height h is compressed by the ambient environment with air pressure p at the so-called barycentric velocity $u_b \equiv \gamma u_1$ and, on the other hand, to the total precipitation power in the condensation area multiplied by $RT = pV$, where V is air molar volume.

If all potential energy of water vapor contained with the condensation area of radius r_1 were instantaneously converted to kinetic energy, it would be concentrated in the rotating windwall of radius r_0. Rotation of the eye at the expense of the kinetic energy of the windwall can be considered as a particular type of dissipation of the kinetic energy of the windwall. Subsequent dissipation of the kinetic energy and angular momentum of the windwall and the eye should occur in the outer environment. For a steady-state hurricane a continuous import of moist air from the environment to the condensation area is necessary to occur at velocity u_1. Local evaporation from the ocean within the condensation area is negligibly small in comparison to condensation rate (1). Hurricane energetics owes itself to condensation of water vapor previously accumulated in the atmosphere. As noted above, the stationary hurricane power can be ensured if the hurricane moves to a new area after the water vapor in the previous area has been depleted.

In conclusion, we note that the pole approximation used in [1] takes into account the major terms r^{-1} and $\ln r$ at $r \ll 1$ and ignores $\partial u/\partial r$, $\ln u$, u_1 and a_{out}. In this case the Bernoulli integral (15) yields algebraic solutions to velocity profiles that agree, to the accuracy of the order of magnitude, with observations for both hurricanes where $r_0 \gg h$ and tornadoes where $h \gg r_0$:

$$u = r\left(\frac{-\ln r - a^2/r^2}{h^2 + r^2}\right)^{1/2}, \qquad w = \frac{h}{r}u, \qquad v = \frac{a}{r}, \quad r \ll 1. \tag{28}$$

The pole approximation allows to estimate the eye radius r_0, maximum velocities and pressure drop. A more detailed solution for the tornado should include consideration of the vertical velocity terms in the Bernoulli integral (15).

References

[1] A.M. Makarieva, V.G. Gorshkov, Phys. Lett. A 373 (2009) 4201.
[2] A.M. Makarieva, V.G. Gorshkov, Int. J. Water 5 (2010), in press.
[3] S.K. Kimball, M.S. Mulekar, J. Clim. 17 (2004) 3555.
[4] R.C. Sheets, J. Appl. Meteor. 8 (1968) 134.
[5] I.-I. Lin, et al., Mon. Wea. Rev. 137 (2009) 3744.
[6] A.M. Makarieva, V.G. Gorshkov, Phys. Lett. A 373 (2009) 2801.
[7] J. Demuth, et al., J. Appl. Meteor. 45 (2006) 1573.
[8] P.J. Michaels, et al., Geophys. Res. Lett. 33 (2006) L09708.
[9] G.J. Holland, Mon. Wea. Rev. 108 (1980) 1212.
[10] G.J. Holland, Mon. Wea. Rev. 136 (2008) 3432.
[11] P.J. Vickery, D. Wadhera, J. Appl. Meteor. Clim. 47 (2008) 2497.
[12] G.M. Dunnavan, J.W. Diercks, Mon. Wea. Rev. 108 (1980) 1915.
[13] J. Callaghan, R.K. Smith, Aust. Met. Mag. 47 (1998) 191.
[14] M.T. Montgomery, et al., Bull. Amer. Met. Soc. 87 (2006) 1335.